XVI.

On the Determination of Transatlantic Longitudes by Means of the Telegraphic Cables.

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After the Telegraphic Cable had been successfully laid between Trinity Bay in Newfoundland and Valencia in Ireland, no time was lost by the Superintendent of the U. S. Coast Survey in making it available for the determination of differences of longitude between the principal meridians of the British Islands and the principal meridians of the United States. The processes and the results of this operation have been fully and ably explained in other publications.[1] In the winter of 1869 – 70, advantage was taken of the French cable, which was then open between Duxbury, Mass., and Brest, France, by the way of the island of St. Pierre, near Newfoundland, to connect, by time-signals, Brest with Duxbury and Duxbury with Cambridge, and hence with Washington. Again, in the summer of 1872, Cambridge exchanged time-signals with St. Pierre by another route, and St. Pierre exchanged signals with Brest by the French cable. Moreover, Mr. J. E. Hilgard, of the U. S. Coast Survey, was despatched to Europe to superintend the exchange of time-signals between Brest and Paris, and between Paris and Greenwich. When all the computations which are required for deducing the final result from the work of the last summer are finished, the Coast Survey Office will possess three independent determinations, by the telegraphic method, of the difference of longitude between Greenwich and Washington. The comparison which has already been made, by anticipation, between the determinations which have been thoroughly computed and those which, as yet, are only known approximately, justifies the expectation that, when the labor of the computers is finished, all the different results will correspond to a surprising degree. The present communication is limited to giving some details in regard to the campaign of 1869 – 70, and the subsequent calculations which grew out of it.

The astronomical station occupied by the U. S. Coast Survey at Brest was under

[1] Smithsonian Contribution to Knowledge, No. 223. Amer. Jour. Sci. N. S. XLIX. p. 228.

the charge of Mr. George W. Dean, of Fall River, Mass., assisted by Mr. F. Blake, experienced officers in the Coast Survey Service. Mr. Dean acknowledges, in his report, the obligations of the Survey to Mr. Thomas Andrews, General Superintendent of the French cable at Brest, for his active and cheerful co-operation in the objects of the expedition, and also to Mr. J. D. H. Dickson for valuable aid in arranging the batteries and other instruments in the cable office.

The U. S. Coast Survey station at Brest is situated near the southeastern part of the grounds attached to the *Établissement des Pupilles de la Marine*, and 126.44 metres west of the flag-staff on the tower of the St. Louis Church, which served as one of the principal points in the primary triangulation of France. Mr. Dean connected this flag-staff with his station by a triangle, with a measured base of 117.33 metres, extending due north from his own position. The Coast Survey Station was distant about one quarter of a mile from the office of the Cable Telegraph, and was temporarily connected with it by two wires. Its latitude is 48° 23′ 21″.4 N.

For the determination of the local time at Brest, U. S. C. S. Transit No. 4, made by Troughton and Simms of London, in 1848, was used. Its principal focus is forty-six inches in length, and its aperture two inches and three quarters of an inch. The power employed was ninety-five. It was noticed that some of the threads, particularly the middle one (D_3), were disturbed by the hygrometric changes of the atmosphere. The level marked (B) was used for ascertaining the inclination of the mechanical axis of the instrument. The arc-value of one division of the scale was found in 1868 to be 0.99 of a second of arc, or 0.066 of a second of time. It was known, by observations made at Salt Lake City in 1869, that the *lamp-end* pivot was the largest, and required a correction amounting to $0^s.013$. The *thread-intervals* were subjected to a new determination, which will be given in another part of this paper.

A Sidereal Chronometer, made by Charles Frodsham of London, and numbered 3451, was loaned to the Coast Survey Service by Professor Joseph Winlock, Director of the Harvard College Observatory. This instrument is provided with a break-circuit, invented by Mr. Frodsham in 1868, so as to record the *alternate* seconds upon a chronograph-sheet; but at the beginning of each minute *two successive* seconds are registered. Mr. Dean observes that this instrument, though its rate was not so uniform at Brest as at the Observatory, has on the whole performed satisfactorily. The Chronograph employed (which was U. S. C. S. No. 1) worked well.

The astronomical station of the U. S. Coast Survey at Duxbury, Mass., was under the charge of Mr. Edward Goodfellow. It is situated on ground belonging to Mr. William Paulding, at a distance of 881 feet E. N. E. from the office of the Cable Tele-

graph. From it the station of Manomet, in the primary triangulation of the Coast Survey, and the secondary station at West Duxbury are both visible. By a preliminary triangulation, resting on a measured base of 380.17 feet, the centre of the Transit-instrument was found to be 6161.73 feet east, and 4861.19 feet north, of the secondary station at West Duxbury. Its latitude is 42° 2′ 53″.1 N. This position had the advantage of an open meridian line to the south of two thousand feet in length. A granite block, with a copper bolt inserted in it, marks the precise spot over which the centre of the Transit-instrument stood.

The observations for instrumental corrections, thread-intervals, and time were made here, as at Brest, according to the chronographic method, and the instruments used were C. S. Transit No. 6, the Krille Sidereal Clock, and U. S. C. S. Chronograph No. 2. The focal length of the Transit-instrument is forty-six inches, and its aperture two inches and three-quarters of an inch. The value of one division of level (B) is equal to 0.86 of a second of arc, or 0.057 of a second of time. The correction to be applied to the lamp-end of the axis for inequality of pivots, as deduced by observations on three different nights, is 0.027 of a second of time, the *lamp-end* being the smallest. The performance of the Krille Clock is reported as tolerably satisfactory, though it appeared to be *under*-compensated. The Chronograph No. 2 had been cleaned and put in good order by the Messrs. Bond before it was taken to Duxbury, but Mr. Goodfellow found that it was difficult to keep it in good adjustment. A fresh determination of the thread-intervals was made at Duxbury. When this instrument was unpacked, it was found that threads (C_4) and (C_5) had become hopelessly twisted together, so that they were useless throughout the campaign.

The observations at Cambridge were under the charge of Professor Joseph Winlock, Director of Harvard College Observatory, those for instrumental corrections and for time being made with the Sidereal Clock and the Chronograph of the Observatory by Mr. E. P. Austin, then an assistant at that place. Coast Survey Transit No. 5 was used. Its position was twenty-eight feet west of the dome of the Observatory, or 0.025 of a second of arc. The value of one division of the level is 0.96 of a second of arc, or 0.064 of a second of time. The *lamp-pivot* is the smallest, and requires a correction of 0.014 of a second of time. No new observations were made for the determination of the thread-intervals. The diaphragm of the Transit-instrument at each of the three stations, Cambridge, Duxbury, and Brest, consists of twenty-five threads, arranged in five tallies.

Whenever the weather permitted, a large set of observations were made *before* and *after* exchanging longitude signals, in order to ascertain the instrumental corrections,

and the error and rate of the clocks or chronometer with the highest degree of accuracy.

In the Coast Survey operations for obtaining differences of longitude between various points in the United States, by the Air Lines of telegraph, it is possible to make the longitude-signal record itself on the chronograph-sheets which register the time, at the station from which the signal goes and also at the station where it is received. But the electrical currents which are practicable with the Cable Lines are too weak to make this record at the latter station. They have barely strength to deflect a delicately suspended magnet, with an attached mirror, from which a beam of light is reflected upon a scale in a dark room. When the signal arrives and this deflection is seen, the observer touches his key and records the moment of its arrival, with as little loss of time as possible, upon the chronograph-sheet.

The longitude signals arranged beforehand between Mr. Dean and Mr. Goodfellow are of four kinds.

I. At the beginning of a minute or any five-seconds mark on a mean-time chronometer or watch, the observer at the first station sends a positive current, of exactly wo seconds' duration, into the cable; at the beginning of the next five-seconds mark he sends a negative current of the same duration, to be followed at the third five-seconds mark by a positive current of equal duration. After an interval of ten seconds succeed four alternating currents in like manner, the first of which is negative. After another interval of ten seconds, three other alternating currents follow, the first of which is negative. These ten signals make what is called a *set*, which may be written thus: —

$$(P^2)\underline{\ 5\ }(N^2)\underline{\ 5\ }(P^2)\underline{\ \ 10\ \ }(N^2)\underline{\ 5\ }(P^2)\underline{\ 5\ }(N^2)\underline{\ 5\ }(P^2)\underline{\ \ 10\ \ }(N^2)\underline{\ 5\ }(P^2)\underline{\ 5\ }(N^2).$$

The object in dividing the set in this way into three groups is for the convenience of identifying the individual signals with facility, when they are recorded upon the chronograph-sheets. After an interval of ten seconds a second set is sent from the same station. In recording these signals at the second station the observer, with his finger upon his chronograph-key, watches closely the bright band, reflected by the mirror on the galvanometer magnet, as it moves towards the *end* of the scale, and at the instant this band darts towards the *centre*, he taps his key, and records upon the chronograph sheet the moment when the cable begins to *discharge* its positive or negative current. After this, two sets of signals are sent in the same way from the second station, and are observed and recorded in a similar manner at the first station. The whole operation

is then repeated, from the first station to the second, and again from the second to the first.

II. The second class of signals differs from the first class in two ways: 1. The positive and negative currents which are sent into the cable are continued for only one half of a second. 2. The observer records the moment when the light begins to move *from* the centre, towards the right or left end of the scale. This class of signals may be written thus:—

$$(P^{\frac{1}{2}})\underline{\;5\;}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\quad 10 \quad}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\;5\;}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\quad 10 \quad}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\;5\;}(N^{\frac{1}{2}}).$$

III. In the third class of signals, alternating positive and negative currents, to the number of six, and each of five seconds' duration, are sent at intervals of ten seconds from the first station, and the moment is recorded at the other station when the light upon the scale darts towards the *centre*. After an interval of fifteen seconds a similar set is sent again. Then the same number of signals are despatched from the second station, to be recorded at the first station. Afterwards, the whole operation is repeated from the first station to the second, and also from the second to the first. This series of twelve signals in two groups may be written thus:—

$$(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5})\underline{\quad 15 \quad}(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5}).$$

IV. The fourth class of signals differs from the third as follows. Four alternating currents make a group, and two groups a set. The duration of each current is ten seconds. The interval between the end of one current and the beginning of the next is five seconds, and the interval between groups is ten seconds, thus:—

$$(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10})\underline{\quad 10 \quad}(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10}).$$

It will be noticed that the signals which are recognized when Classes I., III., and IV. are used, are *discharge* signals; that is, that the moment is recorded when the cessation of the battery current reaches the remote station, and the needle of the galvanometer suddenly returns to the centre. On the other hand, when signals of Class II. are employed, the moment is recorded when the needle begins to show the effect of the *charge*. The velocity with which the signal travels is diminished to a slight extent by the resistance of the battery, and when the sending batteries at the two stations are different, the transmission time derived from currents sent alternately from the two stations will be vitiated to the extent of half this difference in the resistance of

the batteries. With this qualification, this class of signals is useful for the determination of longitude and of the velocity with which the *charge* travels. On the other hand, signals which belong to Classes I., III., and IV. are employed for an exact measurement of the velocity with which the discharge of the battery is propagated through the cable. A preliminary trial, made by Mr. Varley on December 19, revealed the fact that two seconds was insufficient to charge the cable so as to give unequivocal signals, and that the signals of five seconds' duration were better. Accordingly, signals of Class I. were not used at all, and those of Class III. were employed more frequently than those of Class II. The signals of Class IV. were seldom used. Moreover, the original plan laid out beforehand by Mr. Dean was modified so as to make Class II. to comprise six sets of signals, of ten each, and Class III. to comprise six sets of signals, of six each, to be sent in each direction. A good night's work would, therefore, include one hundred and ninety-two signals.

The following data in regard to the Atlantic cable which connects Duxbury with Brest are extracted from the day-book of Mr. Goodfellow. They were communicated to him by R. T. Brown, Esq., Superintendent of the Duxbury Station of the French Transatlantic Cable Company: —

Length of cable from Duxbury to St. Pierre,	749	nautical miles.
Length of cable from St. Pierre to Brest,	2580	" "
Mean cable resistance per knot of 6087 feet, at 75° Fahr., between Duxbury and St. Pierre,	11.99	ohms.
between St. Pierre and Brest,	3.16	"
Mean gutta-percha resistance per knot, at 75° Fahr., between St. Pierre and Brest,	2405	megohms.
between Duxbury and St. Pierre,	2300	"
(when laid and without regard to temperature and pressure.)		
Electrostatic capacity per knot between Duxbury and St. Pierre,	0.358	farads.
between St. Pierre and Brest,	0.404	"
Weight of the conductor per knot between Duxbury and St. Pierre,	107	lbs.
between St. Pierre and Brest,	400	"
Weight of the gutta-percha per knot between Duxbury and St. Pierre,	150	"
between St. Pierre and Brest,	400	"
Diameter of the cores between Duxbury and St. Pierre,	0.282	inch.
between St. Pierre and Brest,	0.468	"
Specific conducting power compared with pure copper, between Duxbury and St. Pierre,	92.9	per cent.
between St. Pierre and Brest,	94.3	" "
Resistance of the galvanometer used in the observations for longitude (temp. of 56° Fahr.),	1188	ohms.
Average distance of the scale from the mirror of the galvanometer,	39	inches.

Value of ten divisions on the scale, ...	0.25 inch.
Number of cells of Minotto's battery used at Duxbury in the longitude campaign, .	40
Number of cells of Minotto's battery used at Brest, ...	30
Electromotive force of the Duxbury battery expressed in Daniell's cells, . .	33.7
Electromotive force of the Brest battery expressed in Daniell's cells, . . .	25.3

On the nights of January 5 and 8, the mirror of the Thomson galvanometer at Duxbury was suspended by a single silk fibre at the top and bottom of the tube. On all other occasions, the mirror was suspended by a single fibre from the top of the tube, the bottom fibre being cut away in order to render the needle more sensitive.

As the Transit room of the Coast Survey at Brest was distant about one quarter of a mile from the cable office, it was necessary to use a Daniell's battery of five cells in the local circuit which connected the key in the cable office with the chronometer and chronograph. At Duxbury, the Transit room and cable office were only one sixth of a mile apart, and a Daniell's battery of three cells was sufficient to work the clock and chronograph circuit.

In order to be able to exchange signals with the Harvard College Observatory, the Coast Survey station at Duxbury was joined to the Boston air-line of telegraph by a loop made of the "American compound telegraph-wire," and with the office of the cable line by a second loop of the same wire, which were kindly loaned by Mr. Moses G. Farmer. This wire is of the diameter known as "steel-core No. 15," its external diameter being No. 13. It weighs one hundred and thirty-four pounds to the mile. Its conductivity is about equal to No. 7 of galvanized iron. The distance from Duxbury to Cambridge by the wire is between forty-three and forty-four miles. The battery used was about fifteen bi-chromate of potash cells. Sometimes, however, thirty-five cells were required, and, at other times, ten cells were sufficient.

The signals used between Cambridge and Duxbury were of two kinds. I. *Clock-signals.* In this case each second break of the clock at one station not only recorded itself upon the chronograph at that station, but, operating upon the main line and the relay magnet at the second station, was also registered by means of the local battery upon the chronograph at the latter station. In sending the signals from Duxbury to Cambridge, the Krille Sidereal Clock was always used. On January 14, Professor Winlock reported that the signals from Duxbury were not well received when his sidereal clock was in the main circuit, so that on and after that date the Mean Time clock, which broke the circuit only every alternate second, was substituted in sending clock-signals to Duxbury. But the times of sending and receiving all kinds of signals were recorded on the chronograph by the sidereal clock. The clock-signals were sent for

one or two minutes at a time. II. The other kind of signals used may be called *hand-signals* or *key-signals*.[1] On December 15, 23, and 31, and on January 3, six sets, of ten breaks each, were sent to Cambridge and received from Cambridge. A few failed of being properly recorded on the chronograph-sheets. On January 14, 22, 24, 26, and 28, and on February 9 and 10, the Cambridge and Duxbury line was connected, by means of a relay-magnet, with the cable key and clock and chronograph circuit at Duxbury, and when the cable signals belonging to Classes II., III., or IV. were received from Brest or sent to Brest, they were recorded on the Duxbury and Cambridge chronograph-sheets. These are called *cable-key* signals, to distinguish them from hand-signals made with a telegraph key at arbitrary times.

After the programme already described had been successfully carried out, with no essential variation, by the officers of the Coast Survey in charge respectively of the three stations, the materials were all placed in my hands by the Superintendent, Professor Benjamin Peirce, in order that I might deduce from them the differences of longitude between Cambridge, Duxbury, and Brest. The computations have been made under my direction, and have been carefully re-examined by me. Those designed to ascertain the clock and instrumental corrections at Cambridge were made by Mr. Henry Gannett. Those necessary to obtain the clock and instrumental corrections at Duxbury and Brest, and also those intended to give a precise determination of the differences of longitude, corrected for the Personal Equation in observing transits and noting cable signals, were made by Mr. Lucius Brown. I shall now describe the methods of computation which have been followed.

The plan which has been adopted for the precise determination of the local time at each of the three stations is essentially the same as that upon which F. G. W. Struve proceeded in working up the results of his two chronometric expeditions to ascertain the differences of longitude between Pulkowa and Altona, and between Altona and Greenwich, and which has become familiar in the Coast Survey Service by the labors of the late Mr. Sears C. Walker and Dr. B. A. Gould. I have not thought that observations made with small portable Transit-instruments, such as those which are used at the Coast Survey stations, would be accurate enough to justify the added labor of assigning different weights to the stars observed for time, according to their declination, after the manner indicated by Struve and J. A. C. Oudemans.[2]

[1] These are sometimes called *arbitrary* signals, to distinguish them from *clock* signals.

[2] Dissertatio Astronomica Inauguralis.

Formulæ used in the Computation of Instrumental and Clock Corrections.

α	$= T + \Delta T + \tau$ (τ being the correction *in time* for errors in azimuth, level, inequality of pivots, and collimation.)
τ	$= a\,\dfrac{\sin(\phi-\delta)}{\cos\delta} + b\,\dfrac{\cos(\phi-\delta)}{\cos\delta} \pm \dfrac{c}{\cos\delta} = Aa + Bb^0 \pm Cc$,* (a) being the azimuth constant, (b) being the level constant, and (c) being the collimation constant.
ϕ	= the geographical latitude of the station.
α	= the right ascension of the star.
δ	= the declination of the star.
M	= the mean of the observed tallies.
f	= the mean of the equatorial thread-intervals.
ρ	= the correction for rate, and its log is 0.000005 for a gain of 1 sec. daily: 0.00119 for a mean time clock.
σ	= the sine correction when star is near the pole: log σ being additive to log F; when F sec δ is less than 2^{m} it may be neglected.
R	$= F$ sec $\delta.\ \sigma.\ \rho$; and is to be added to M to obtain the time of transit over the mean of all the threads.
b_0	= the level constant in time, corrected for inequality of pivots: it is positive for W. end high.
Star constant A	$= \dfrac{\sin(\phi-\delta)}{\cos\delta}$.
Star constant B	$= \dfrac{\cos(\phi-\delta)}{\cos\delta}$.
Star constant C	= sec δ.
	$180° - \delta$ is used instead of δ when the star is below the pole.
	A is positive, except for stars between the Zenith and North Pole.
	B is positive, except for stars at lower culmination.
	C is positive, except for stars at lower culmination.
k	= the diurnal aberration $= 0^{\mathrm{s}}.021 \cos\phi \sec\delta$. It is (—) in upper, (+) in lower culminations.
Bb_0	= correction for level and inequality of pivots.
T	$= M + R =$ time of transit over the mean of all the threads.
t	$= T + Bb_0 + \kappa$.
ΔT	$= \Delta t =$ correction of the clock.
ω	$= \alpha - t = \Delta t + Aa \pm Cc$.
Cc	= correction for collimation.
Aa	= correction for azimuth.
ω_0	$= \omega \mp Cc$, upper sign for lamp west.

The collimation constant (c) is determined from reversals on circumpolar stars, and is to be obtained from the equation

$$t_e - t_w = \omega_w - \omega_e = 2\,Cc.$$

If the collimation is known, and the corresponding corrections are applied, reduce the value of ω_0 for the several stars to an arbitrary time T_0, by applying the correction for daily rate. Calling this reduced value (ω_0),

$$(\omega_0) = \omega_0\,\frac{t - T_0}{24\text{ hrs.}} \times \text{daily rate.}$$

* Mayer's formula, corrected for inequality of pivots and aberration.

The local time and azimuth are obtained thus: Assume an approximate value of the clock correction $= \theta$ for the arbitrary time T_0, and make $(\omega_0) - \theta = \omega_0'$.

For each star, $\varDelta t + A a = (\omega_0) = \omega_0' + \theta$.

If $\varDelta t - \theta = \varDelta \theta$ we have $A a + \varDelta \theta = \omega_0'$, ω_0' being a small residual.

Following the method of least squares, multiply this equation for each star by the coefficients of the unknown quantities, and we obtain the normal equations: —

$$\text{(1.)} \qquad \Sigma A a + \Sigma \varDelta \theta = \Sigma \omega_0'.$$

$$\text{(2.)} \qquad \Sigma A^2 a + \Sigma A \varDelta \theta = \Sigma A \omega_0'.$$

From these two equations, we can compute the values of a, $\varDelta \theta$, and hence derive the value of $\varDelta t$ for the time T_0.

When the collimation constant has not been determined by reversals, but enters into the equations as one of the unknown quantities, we have for each star

$$A a \pm C c + \varDelta \theta = \omega',$$

(ω' being equal to $(\omega) - \theta$). In this case the normal equations are: —

$$\text{(1.)} \qquad \Sigma A a \pm \Sigma C c + \Sigma \varDelta \theta = \Sigma \omega.$$

$$\text{(2.)} \qquad \Sigma A^2 a \pm \Sigma A C c + \Sigma A \varDelta \theta = \Sigma A \omega'.$$

$$\text{(3.)} \qquad \Sigma A C a \pm \Sigma C^2 c + \Sigma C \varDelta \theta = \Sigma C \omega'.$$

In the computations for Brest and Duxbury, only two normal equations were used, the value of (c) being assumed from the result of the reversals. In the computations for Cambridge, the value of (c) was assumed from reversals, for December 17 and 31, for January 3, and for February 1. On January 4 the value of (c) was taken from January 3. On January 12, the values calculated for January 11 and 16 were adopted. On January 19, the values calculated for January 18 and 26 were adopted. On February 11, the value calculated for February 10 was adopted. On the other nights (c) was calculated by means of the three normal equations.

Adopted Values of the Equatorial Thread Intervals.

	C. S. Transit Instrument, No. 4. From 31 Observations.		C. S. Transit Instrument, No. 5. From 24 Observations.		C. S. Transit Instrument. No. 6. From 38 Observations.
B_1	+34.174	B_1	+36.450	B_1	+35.590
B_2	+31.817	B_2	+33.924	B_2	+33.094
B_3	+29.269	B_3	+31.320	B_3	+30.607
B_4	+26.860	B_4	+28.800	B_4	+28.038
B_5	+24.308	B_5	+26.202	B_5	+25.403
C_1	+19.433	C_1	+20.950	C_1	
C_2	+17.173	C_2	+18.307	C_2	
C_3	+14.533	C_3	+15.691	C_3	+15.399
C_4	+12.019	C_4	+13.051	C_4	+12.693
C_5	+ 9.826	C_5	+10.498	C_5	+10.239
D_1	+ 5.035	D_1	+ 5.232	D_1	+ 5.107
D_2	+ 2.484	D_2	+ 2.551	D_2	+ 2.589
D_3	— 0.043	D_3	— 0.082	D_3	+ 0.099
D_4	— 2.364	D_4	— 2.630	D_4	— 2.450
D_5	— 4.755	D_5	— 5.271	D_5	— 5.059
E_1	— 9.691	E_1	—10.413	E_1	—10.071
E_2	—12.205	E_2	—12.972	E_2	—12.821
E_3	—14.630	E_3	—15.565	E_3	—15.331
E_4	—17.153	E_4	—18.201	E_4	—17.950
E_5	—19.470	E_5	—20.787	E_5	—20.440
F_1	—24.451	F_1	—26.102	F_1	—25.504
F_2	—26.818	F_2	—28.671	F_2	—28.073
F_3	—29.399	F_3	—31.522	F_3	—30.660
F_4	—31.799	F_4	—34.057	F_4	—33.107
F_5	—34.153	F_5	—36.703	F_5	—35.782

The equatorial thread-intervals used in the computations were obtained for the C. S. transit-instruments No. 4 and No. 6 from the observations made by Mr. Dean and Mr. Goodfellow in the course of the campaign. The equatorial thread-intervals for C. S. transit-instrument No. 5 are the same as those found by Mr. Isaac Bradford, from the observations made at Cambridge, in 1869, by Mr. A. F. Mosman, and Mr. F. Blake, Jr. No observations were made for this purpose in the campaign of 1869–70.

The computations to find the clock and instrumental corrections, for a *single* night, are given in detail, as an illustration of the method adopted.

Computation of Transits observed at Cambridge on January 3, 1870.

Star	η Piscium.	o Piscium.	β Arietis.	50 Cassiop.	i Cassiop.	i Cassiop.
Lamp	W.	W.	W.	W.	W.	E
Threads	C D E	C D E	C D E	B* C D	B C	B C
α	$1^h\ 24^m\ 31^s.11$	$1^h\ 38^m\ 31^s.35$	$1^h\ 47^m\ 27^s.27$	$1^h\ 52^m\ 22^s.63$	$2^h\ 18^m\ 23^s.72$	$2^h\ 18^m\ 23^s.72$
δ	14° 40′ 26″	8° 30′ 3″	20° 10′ 16″	71° 47′ 35″	66° 49′ 5″	66° 49′ 5″
M	$26^m\ 28^s.887$	$40^m\ 28^s.853$	$49^m\ 24^s.946$	$53^m\ 36^s.956$	$19^m\ 23^s.015$	$21^m\ 23^s.541$
f	+ .024	+ .024	+ .024	+ 14.362	+ 23.519	— 23.519
log. f	8.38021	8.38021	8.38021	1.15721	1.37142	1.37142n
secant δ	.01441	.00480	.02749	.50522	.40488	
log. ϱ						
log. σ						
log. R	8.39462	8.38501	8.40770	1.66243	1.77630	1.77630n
R	.025	.024	.026	45.965	59.744	— 59.744
b_0 or b_1	— .136	— .150	— .159	— .162	— .186	— .116
A	.481	.564	.403	— 1.572	— 1.051	
B	.916	.839	.986	2.788	2.313	
C	1.034	1.011	1.065	3.200	2.540	
k	— .016	— .016	— .016	— .050	— .039	— .039
Bb_0 or Bb_1	— .125	— .126	— .157	— .452	— .430	— .268
T	$26^m\ 28^s.912$	$40^m\ 28^s.877$	$49^m\ 24^s.972$	$54^m\ 22^s.921$	$20^m\ 22^s.759$	$20^m\ 23^s.797$
t	26 28,771	40 28.735	49 24.799	54 22.419	20 22.290	20 23.490
ω	1 57.661	1 57.385	1 57.529	1 58.789		
Cc	.242	.237	.249	.749		
ω_0	1 57.903	1 57.622	1 57.778	1 59.538		
(ω_0)	1 57.888	1 57.609	1 57.766	1 59.527		

* The observation on thread B_2 was lost.

Computation of Transits observed at Cambridge on January 3, 1870.

Star	5 Urs. Min.	γ Ceti.	β Urs. Min.	ζ Arietis.	α Persei.	γ^2 Urs. Min.	δ Persei.
Lamp	E. (L. C.)	E.	E. (L. C.)	E.	E.	E. (L. C.)	E.
Threads	C D E	C D E	D_5 E F	C D E	C D E	C D E	C D E
α	$2^h\ 27^m\ 46^s.43$	$2^h\ 36^m\ 33^s.73$	$2^h\ 51^m\ 3^s.23$	$3^h\ 7^m\ 25^s.89$	$3^h\ 15^m\ 3^s.42$	$3^h\ 20^m\ 53^s.87$	$3^h\ 33^m\ 40^s.91$
δ	76° 16′ 4″	2° 41′ 3″	74° 41′ 1″	20° 33′ 37″	49° 23′ 49″	72° 17′ 40″	47° 22′ 11″
M	$29^m\ 39^s.371$	$38^m\ 31^s.695$	$54^m\ 19^s.845$	$9^m\ 24^s.080$	$17^m\ 2^s.091$	$22^m\ 47^s.800$	$35^m\ 39^s.640$
f	+ .024	− .024	− 21.842	− .024	− .024	+ .024	− .024
log. f	8.38021	8.38021n	1.33929n	8.38021n	8.38021n	8.38021n	8.38021n
secant δ	.62462	.00049	.57814	.02858	.18654	.51693	.16925
log. ϱ							
log. σ							
log. R	9.00483	8.38069n	1.91743n	8.40879n	8.56675n	8.89714	8.54946n
R	.101	− .024	−1 22.686	− .026	− .037	+ .079	− .035
b_0 or b_1	− .126	− .149	− .191	− .221	− .244	− .257	− .214
A	3.697	.639	3.371	.397	− .188	2.988	− .128
B	− 2.020	.770	− 1.722	.992	1.525	− 1.373	1.470
C	− 4.213	1.001	− 3.786	1.068	1.537	− 3.288	1.476
k	+ .066	− .016	+ .058	− .017	− .024	+ .051	− .023
Bb_0 or Bb_1	.255	− .115	+ .329	− .219	− .372	+ .352	− .315
T	$29^m.39^s.472$	$38^m\ 31^s.671$	$52^m\ 57^s.159$	$9^m\ 24^s.054$	$17^m\ 2^s.054$	$22^m\ 47^s.879$	$35^m\ 39^s.505$
t	29 39.793	38 31.540	52 57.546	9 23.818	17 1.658	22 48.282	35 39.167
ω	1 53.363	1 57.810	1 54.316	1 57.928	1 58.248	1 54.412	1 58.267
Cc	+ .986	− .234	+ .886	− .250	− .359	+ .769	− .345
ω_0	1 54.349	1 57.576	1 55.202	1 57.678	1 57.889	1 55.181	1 57.922
(ω_0)	1 54.342	1 57.570	1 55.197	1 57.676	1 57.887	1 55.180	1 57.922

Computations of Transits observed at Cambridge on January 3, 1870.

Star	η Tauri.	ζ Persei.	α Camelop.	*i* Aurigæ.	11 Orionis.	α Aurigæ.
Lamp	E.	E.	E.	E.	W.	W.
Threads	C D E	D E F	C D E	C D E	C D E	B C D
α	$3^h\ 39^m\ 45^s.67$	$3^h\ 45^m\ 58^s.02$	$4^h\ 41^m\ 9^s.86$	$4^h\ 48^m\ 32^s.18$	$4^h\ 57^m\ 8^s.87$	$5^h\ 7^m\ 5^s.92$
δ	23° 42′ 1″	31° 29′ 41″	66° 7′ 6″	32° 57′ 25″	15° 13′ 9″	45° 51′ 44″
M	$41^m\ 43^s.941$	$47^m\ 38^s.027$	$43^m.\ 9^s.715$	$50^m\ 30^s.547$	$59^m\ 6^s.502$	$8^m\ 41^s.248$
f	— .024	+ 15.680	— .024	— .024	+ .024	13.666
log. f	8.38021n	1.19535	8.38021n	8.38021n	8.38021	1.19496
secant δ	.03826	.06921	.39272	.07619	.01551	.15715
log. ϱ						
log. σ						
log. R	8.41847n	1.26456	8.77293n	8.45640n	8.39572	1.35211
R	— .026	+ 18.389	— .059	— .029	+ .025	+ 22.491
b_0 or b_1	— .188	— .173	— .249	— .268	— .152	— .141
A	.350	.223	— .994	— .195	.473	— .087
B	1.035	1.152	2.261	1.176	.922	1.433
C	1.092	1.173	2.470	1.192	1.036	1.436
k	— .017	— .018	— .038	— .018	— .016	— .022
Bb_0 or Bb_1	— .195	— .199	— .563	— .315	— .140	— .202
T	$41^m\ 43^s.915$	$47^m\ 56^s.416$	$43^m\ 9^s.656$	$50^m\ 30^s.518$	$56^m\ 6^s.527$	$9^m\ 3^s.739$
t	41 43.703	47 56.199	43 9.055	50 30.185	56 6.371	9 3.515
ω	1 58.033	1 58.179	1 59.195	1 58.005	1 57.501	1 57.595
Cc	— .253	— .274	— .578	— .279	+ .242	+ .336
ω_0	1 57.780	1 57.905	1 58.617	1 57.726	1 57.743	1 57.931
(ω_0)	1 57.780	1 57.907	1 58.629	1 57.735	1 57.753	1 57.943

Computation of Transits observed at Cambridge on January 3, 1870.

Star	β Orionis.	β Tauri.	Groomb. 966	δ Orionis.	α Leporis.	ε Orionis.	ω Draconis.
Lamp	W.	W.	W.	W.	W.	W.	W. (L. C.)
Threads	E* F	C D E	C D E	C D E	C D E	C D E	D E F
α	$5^h\ 8^m\ 17^s.87$	$5^h\ 18^m\ 4^s.90$	$5^h\ 22^m\ 24^s.48$	$5^h\ 25^m\ 22^s.38$	$5^h\ 27^m\ 0^s.45$	$5^h\ 29^m\ 37^s.47$	$5^h\ 37^m\ 39^s.35$
δ	S 8° 21′ 23″	28° 29′ 38″	74° 57′ 6″	S 0° 23′ 59″	S 17° 55′ 11″	S 1° 17′ 21″	68° 49′ 1″
M	$10^m\ 40^s.516$	$20^m\ 2^s.735$	$24^m\ 23^s.559$	$27^m\ 19^s.803$	$28^m\ 57^s.563$	$31^m\ 34^s.885$	$38^m\ 52^s.055$
f	− 24.953	+ .024	+ .024	+ .024	+ .024	+ .024	− 15.680
log. f	1.39712n	8.38021	8.38021	8.38021	8.38021	8.38021	1.19535n
secant δ	.00464	.05607	.58565	.00001	.02160	.00011	.44208
log. ϱ							
log. σ							
log. R	1.40176n	8.43628	8.96586	8.38022	8.40181	8.38032	1.63743
R	− 25.220	+ .027	+ .092	+ .024	+ .025	+ .024	+ 43.394
b_0 or b_1	− .139	− .128	− .124	− .121	− .120	− .117	− .110
A	.783	.273	− 2.074	.679	.913	.691	2.580
B	.640	1.105	3.246	.734	.521	.724	− 1.001
C	1.011	1.138	3.852	1.000	1.051	1.000	− 2.767
k	− .016	− .018	− .060	− .016	− .016	− .016	+ .043
Bb_0 or Bb_1	− .089	− .141	− .403	− .089	− .063	− .085	+ .110
T	$10^m\ 15^s.296$	$20^m\ 2^s.762$	$24^m\ 23^s.651$	$27^m\ 19^s.827$	$28^m\ 57^s.588$	$31^m\ 34^s.909$	$39^m\ 35^s.449$
t	10 15.191	24 2.603	24 23.188	28 19.722	28 57.509	31 34.808	39 35.602
ω	1 57.321	1 57.703	1 58.708	1 57.342	1 57.059	1 57.338	1 56.252
Cc	+ .237	+ .266	+ .901	+ .234	+ .246	+ .234	− .647
ω_0	1 57.558	1 57.969	1 59.609	1 57.576	1 57.305	1 57.572	1 55.605
(ω_0)	1 57.570	1 57.982	1 59.722	1 57.590	1 57.319	1 57.586	1 55.620

* The observation on E_1 was lost.

Computations for the Error of the Clock and for Instrumental Corrections, January 3, 1873.

Star.	Lamp.	(ω_0)	A	(ω_0')	A^2	$(A\omega_0')$	$A\alpha$	$\varDelta T_0$	Residuals.	(Residuals)^2
		m s						m s		
η Piscium	W.	1 57.888	.481	— .012	.231	— .006	.436	1 58.324	.275	.076
o Piscium	W.	1 57.609	.564	— .291	.318	— .164	.511	1 58.120	.071	.005
β Arietis	W.	1 57.766	.403	— .134	.162	— .058	.366	1 58.132	.083	.007
50 Cassiop.	W.	1 59.527	— 1.572	1.627	2.470	— 2.558	—1.425	1 58.102	.053	.003
5 Urs. M. (L. C.)	E.	1 54.342	3.697	—3.558	13.669	—13.134	3.352	1 57.695	—.354	.125
γ Ceti	E.	1 57.570	.639	— .330	.409	— .211	.580	1 58.150	.101	.010
β Urs. M. (L. C.)	E.	1 55.197	3.371	—2.703	11.364	— 9.112	3.057	1 58.254	.185	.034
ζ Arietis	E.	1 57.676	.397	— .224	.157	— .149	.261	1 57.937	—.112	.013
α Persei	E.	1 57.887	— .188	— .013	.035	+ .002	— .171	1 57.716	—.333	.111
γ^2 Urs. M. (L. C.)	E.	1 55.180	2.988	—2.720	8.927	— 8.127	2.710	1 57.890	—.159	.025
δ Persei	E.	1 57.922	— .128	.022	.016	— .003	— .115	1 57.807	—.242	.059
η Tauri	E.	1 57.781	.350	— .119	.122	— .042	.317	1 58.098	.049	.002
ζ Persei	E.	1 57.907	.223	.007	.049	+ .002	.201	1 58.108	.059	.003
α Camelop.	E.	1 58.629	— .994	.929	.989	— .923	— .900	1 57.929	—.120	.001
ι Aurigæ	E.	1 57.735	.195	— .165	.038	— .032	.176	1 57.911	—.138	.019
11 Orionis	W.	1 57.753	.473	— .147	.224	— .070	.429	1 58.182	.133	.018
α Aurigæ	W.	1 57.943	— .087	.043	.006	— .004	— .078	1 57.865	—.184	.034
β Orionis	W.	1 57.570	.783	— .330	.612	— .026	.709	1 58.279	.230	.053
β Tauri	W.	1 57.982	.273	.082	.075	+ .022	.248	1 58.230	.181	.033
Groomb. 966	W.	1 59.722	— 2.074	1.822	4.300	— 3.779	—1.821	1 57.901	—.148	.022
δ Orionis	W.	1 57.590	.679	— .310	.461	— .210	.615	1 58.205	.156	.024
α Leporis	W.	1 57.319	.913	— .581	.833	— .530	.828	1 58.147	.098	.010
ε Orionis	W.	1 57.586	.691	— .314	.477	— .217	.626	1 58.212	.163	.027
ω Draco. (L. C.)	W.	1 55.620	2.580	—2.280	6.656	— 5.884	2.340	1 57.969	—.080	.006
			+14.757	—9.699	+52.610	—45.445				

Collimation Constant.

Star.	t	Lamp.	$2\,Cc$	c
ι Cassiopeæ	$20^m\ 22^s.290$	West	+1.200	+.236
	20 23.490	East		

$$t_e - t_w = w_w - w_e = 2\,Cc$$

$$c = \frac{t_e - t_w}{2\,C}$$

Hourly rate of clock $= -\ 0^s.007$

Normal Equations for the weight of $\varDelta\theta$.

$$\varDelta\theta + .616\,a - 1 = 0$$
$$14.757\ \varDelta\theta + 52.610\,a = 0$$
$$p \text{ (or weight of } \varDelta\theta) = 6.41$$
$$\epsilon = \frac{(\text{Residuals})^2}{22} = {}^s.181$$
$$r = .6745\,\epsilon = {}^s.122$$
$$r_0 = \frac{r}{\sqrt{p}} = {}^s.043$$

Normal Equations.

$$\Sigma\varDelta\theta + \Sigma A a + \Sigma\omega_0 = 0$$
$$24\,\varDelta\theta + 14.757\,a - 9.699 = 0$$
$$\varDelta\theta + .616\,a - .404 = 0$$
$$\Sigma A\,\varDelta\theta + \Sigma A^2 a + \Sigma A\,\omega_0' = 0$$
$$14.757\ \varDelta\theta + 52.610\,a - 45.445 = 0$$
$$14.757\ \varDelta\theta + \ 9.076\,a - \ 5.965 = 0$$
$$43.534\,a - 39.480 = 0$$
$$a = +\,.907$$

$$a = +\,.907$$
$$\varDelta\theta = -\,.150$$
$$\varDelta T_0 = -\ 1^m\ 58^s.050$$
$$\theta = -\ 1^m\ 57^s.900$$
$$T_0 = 3^h\ 30^m$$

The probable error of $\varDelta\theta$ and of $\varDelta T_0$ is $\pm\ {}^s.043$.

In the following tables, the first column contains the dates, the second (a) the correction for the azimuth of the transit instrument, the third (c) its correction for collimation, the fourth ($\varDelta T_0$) the error of the clock or chronometer, the fifth the hourly rate of the clock or chronometer, and the sixth the number of stars of which the transits were recorded on the corresponding date.

CAMBRIDGE.

Date.	a	c	$\varDelta T_0$	Hourly Rate.	Number of Stars.
1869.	s	s	m s	s	
Dec. 14	+1.797	+0.220	2 0.695	+0.029	7
" 15	+1.863	+0.154	2 0.212	+0.019	12
" 17	+0.794	0.198	1 59.513	+0.007	14
" 23	+0.674	0.241	1 58.962	+0.004	18
" 31	+0.840	0.258	1 58.402	+0.005	18
1870.					
Jan. 3	+0.907	+0.236	1 58.054	—0.007	24
" 4	+0.223	+0.236	1 57.867		2
" 7	+1.047	+0.312	1 57.392	+0.013	8
" 11	+0.846	+0.234	1 55.618	+0.020	11
" 12	+0.603	+0.276	1 55.096	+0.019	4
" 16	+0.978	+0.257	1 53.636	+0.016	21
" 18	+0.878	+0.259	1 52.844	+0.021	13
" 19	+0.839	+0.258	1 52.243	+0.022	8
" 26	+0.915	+0.254	1 49.744	+0.012	15
" 28	+0.890	+0.216	1 49.316	+0.013	16
Feb. 1	+0.853	+0.311	1 47.806	—0.021	4
" 10	+0.827	+0.235	1 42.430	+0.024	15
" 11	+0.796	0.235	1 41.850	+0.025	8

BREST.

Date.	a	c	$\varDelta T_0$	Hourly Rate.	Number of Stars.
1870.	s	s	s s	s	
Jan. 5	+0.299	—0.004	— 3.703 ± 0.039	+0.067	12
" 5	+0.286	—0.004	— 3.810 ± 0.033	+0.067	11
" 8	+0.266	+0.0005	— 6.899 ± 0.034	+0.033	9
" 9	+0.145	+0.0005	— 7.474 ± 0.015	+0.015	15
" 9	+0.223	+0.0005	— 7.627 ± 0.029	+0.015	6
" 10	—0.507	—0.005	— 7.533 ± 0.016	+0.004	15
" 14	+0.426	—0.048	— 9.009 ± 0.030	+0.030	18
" 17	+0.057	—0.003	—11.447 ± 0.018	—0.004	19
" 17	+0.027	—0.003	—11.408 ± 0.019	—0.004	6
" 22	+0.229	+0.010	— 6.685 ± 0.022	—0.030	17
" 24	+0.244	—0.032	— 5.180 ± 0.016	—0.048	14
" 26	+0.433	—0.020	— 2.125 ± 0.016	—0.070	14
" 26	+0.426	—0.020	— 1.824 ± 0.020	—0.070	6
" 28	+0.504	+0.073	+ 1.218 ± 0.022	—0.060	13
" 28	+0.488	+0.023	+ 1.454 ± 0.040	—0.060	6
" 29	+0.473	—0.015	+ 2.453 ± 0.017	—0.030	14
Feb. 10	+0.785	+0.002	— 0.020 ± 0.009	—0.012	9
" 10	+0.888	+0.002	+ 0.026 ± 0.022	—0.012	12

DUXBURY.

Date.	a	c	ΔT_0	Hourly Rate.	Number of Stars.
1869.	s	s	s s	s	
Dec. 14	—0.367	+0.472	+13.548 ± 0.042	—0.160	5
" 14	—0.262	+0.472	+13.868 ± 0.026	—0.160	8
" 15	+0.278	—0.261	+ 2.482 ± 0.021	—0.180	10
" 15	+0.168		+ 3.132 ± 0.031		7
" 23	—0.241	—0.015	— 2.343 ± 0.013	—0.040	19
" 23	—0.320	—0.015	— 2.143 ± 0.016	—0.040	11
" 31	+0.087	—0.015	+ 4.328 ± 0.015	—0.030	13
" 31	+0.081	—0.015	+ 4.361 ± 0.024	—0.030	7
1870.					
Jan. 3	+0.232	—0.010	+ 6.627 ± 0.019	—0.030	16
" 3	+0.122	—0.010	+ 6.806 ± 0.021	—0.030	8
" 4	—0.098	—0.032	+ 7.433 ± 0.015	—0.025	9
" 5	+0.038	—0.023	+ 7.786 ± 0.032	0.000	4
" 5	—0.184		+ 7.951 ± 0.030	—0.045	10
" 7	+0.117	—0.034	+ 7.532 ± 0.027	+0.010	7
" 8	+0.089	—0.051	+ 7.343 ± 0.015	+0.045	11
" 9	+0.478	—0.076	+ 6.584 ± 0.020	+0.012	10
" 11	+0.180	—0.046	+ 6.607 ± 0.025	+0.000	18
" 16	—0.121	—0.037	+ 6.163 ± 0.026	+0.000	9
" 18	+0.013	—0.151	+ 9.462 ± 0.034	—0.015	7
" 19	+4.234	—0.198	+10.026 ± 0.023	—0.018	17
" 21		—0.035		—0.030	
" 22	—0.340	—0.065	+11.638 ± 0.023	—0.030	6
" 23	—0.139	—0.074	+12.420 ± 0.027	—0.040	3
" 26	+0.309	—0.100	+15.579 ± 0.022	—0.045	12
" 26	+0.082		+16.092 ± 0.016		10
" 28	+0.102	—0.085	+18.019 ± 0.018	—0.040	9
" 28	—0.192		+18.453 ± 0.016		8
Feb. 10	—0.196	—0.001	— 2.256 ± 0.021	0.000	6
" 10	—0.373	—0.001	— 2.117 ± 0.024	0.000	7
" 11	—0.015	—0.000	— 2.298 ± 0.026	0.000	4

After the instrumental and the clock corrections have been found, the difference of longitude is computed in the following manner: —

T_1 is the *mean* of the recorded Brest clock times of sending a *set* of westward signals.

T_1' is the *mean* of the recorded Duxbury clock times of receiving these signals.

T_2 is the *mean* of the recorded Brest clock times of receiving a *set* of eastward signals.

T_2' is the *mean* of the recorded Duxbury clock times of sending these signals.

$T_1 - T_1'$ is the mean difference for the westward set.

$T_2 - T_2'$ is the mean difference for the eastward set.

$\Delta T_1 - \Delta T_1'$ is the difference of the clock corrections at Brest and Duxbury, when the westward signals were sent.

$\Delta T_2 - \Delta T_2'$ is the difference of the clock corrections at Brest and Duxbury, when the eastward signals were sent.

x is the mean transmission-time of signals.

λ is the difference of longitude.

λ and x are computed by these two formulas.

(1.) $$2\lambda = (T_1 - T_1') + (T_2 - T_2') + (\Delta T_1 - \Delta T_1') + (\Delta T_2 - \Delta T_2')$$

(2.) $$2x = (T_2 - T_2') - (T_1 - T_1') + (\Delta T_2 - \Delta T_2') - (\Delta T_1 - \Delta T_1')$$

When signals are sent in only one direction,

(3.) $$\lambda = (T_1 - T_1') + (\Delta T_1 - \Delta T_1') + x, \quad \text{or}$$

(4.) $$\lambda = (T_2 - T_2') + (\Delta T_2 - \Delta T_2') - x$$

In using the formulas (3) and (4), we must adopt an assumed value of x, and this assumed value may be the one computed from the perfect sets of signals of the same class exchanged on the *same* night, or the more general mean derived from the perfect sets exchanged on *all* the nights.

The following table contains the differences of longitude and the transmission time between Duxbury and Brest, *not* corrected for personal equation.

		Class II.		Class III.	
Date.	Sets.	λ	x	λ	x
Jan. 5	1			h m s 4 24 42.731	s 1.135
" "	2			42.762	1.088
" "	Mean			42.746	1.111
Jan. 8	1	h m s 4 24 42.910	s 1.223	s 42.852	s 1.089
" "	2	42.888	1.227	42.781	1.123
" "	3	42.874	1.187	42.968	1.143
" "	4	42.875	1.167	42.950	1.140
" "	5	42.896	1.103	42.838	1.151
" "	6	42.916	1.140	42.888	1.116
" "	Mean	42.893	1.174	42.879	1.127
Jan. 9	1	s 42.891	s 1.052	s 42.854	s 1.055
" "	2	42.889	1.090	42.825	1.053
" "	3	42.850	1.098	42.844	1.063
" "	4	42.886	1.105	42.840	1.062
" "	5	42.923	1.133	42.895	1.001
" "	6	42.934	1.119	42.889	1.031
" "	Mean	42.895	1.100	42.858	1.044
Jan. 10	1	s 42.512	s 1.132	s 42.503	s 1.052
" "	2	42.494	1.126	42.479	1.079
" "	3	42.520	1.147	42.509	1.045
" "	4	42.496	1.114	42.479	1.057
" "	5	42.523	1.116	42.502	1.054
" "	6	42.546	1.139	42.515	1.050
" "	Mean	42.515	1.129	42.498	1.056
Jan. 14	1	s 42.002	s 1.083	s 42.115	s 1.168
" "	2	42.024	1.107	42.070	1.132
" "	3	42.002	1.039	42.067	1.087
" "	4	42.991	1.114	42.063	1.073
" "	5	42.018	1.079	42.071	1.128
" "	6	42.979	1.087	42.011	1.098
" "	Mean	42.003	1.085	42.066	1.114
Jan. 17	1	s 42.870	s 1.064	s 42.998	s 1.131
" "	2	42.873	1.064	42.998	1.251
" "	3	42.880	1.060	42.994	1.195
" "	4	42.922	1.057	42.903	1.107
" "	5	42.904	1.065	42.989	1.146
" "	6	42.900	1.070	42.982	1.174
" "	Mean	42.892	1.063	42.977	1.167

	Class II.			Class III.	
Date.	Sets.	λ	x	λ	x
		s	s	s	s
Jan. 22	1	42.811*		42.937	1.105
“ “	2	42.859*		43.016	1.132
“ “	3	42.930*		43.022	1.145
“ “	4	42.893*		43.040	1.168
“ “	5	42.825	1.170	43.005	1.188
“ “	6	42.856	1.139	43.078	1.244
“ “	Mean	42.853	1.154	43.016	1.164
		s	s	s	s
Jan. 24	1	43.018	1.152	43.130*	
“ “	2	43.009	1.096	43.135	1.383
“ “	3	43.067	1.090	43.115	1.314
“ “	4	43.073	1.092	43.170	1.310
“ “	5	43.037	1.110	43.162	1.285
“ “	6	43.023	1.156	43.163	1.249
“ “	Mean	43.038	1.116	43.148	1.308
		s	s	s	s
Jan. 26	1	43.080*		43.246	1.176
“ “	2	43.077	1.065	43.196	1.144
“ “	3	43.122	1.106	43.186	1.170
“ “	4	43.102	1.098	43.181	1.131
“ “	5	43.081*		43.154	1.187
“ “	6	43.145*		43.166	1.134
“ “	Mean	43.101	1.090	43.188	1.157
		s	s	s	s
Jan. 28	1	43.151	1.091	43.310*	
“ “	2	43.160	1.085	43.296	1.200
“ “	3	43.143	1.098	43.325	1.183
“ “	4	43.168	1.072	43.263	1.182
“ “	5	43.151	1.070	43.226	1.135
“ “	6	43.141	1.069	43.259	1.170
“ “	Mean	43.152	1.081	43.276	1.174
		s		s	s
Feb. 9	1	43.098		43.076*	
“ “	2	43.084		43.070*	
“ “	3	43.085		43.130	1.228
“ “	4	43.096		43.048	1.206
“ “	5	43.121		43.048*	
“ “	6	43.146		43.028	1.216
“ “	Mean	43.105		43.068	1.217

	Class II.			Class IV.	
Date.	Sets.	λ	x	λ	x
		s		s	
Feb. 10	1	42.683			
“ “	2	42.629		42.743	
“ “	3	42.625		42.652	
“ “	4	42.645		42.715	
“ “	5			42.610	
“ “	6			42.671	
“ “	7			42.630	
“ “	8			42.617	
“ “	Mean	42.645		42.663	

A weight of $\frac{1}{3}$ is given to each of the values marked with an asterisk in obtaining the means of the groups. For these values were obtained from signals in one direction only, with assumed values of the transmission time, derived from other sets of the same group. On February 10, the Brest signals were not received at Duxbury. An assumed value of (x) was applied to the signals of Class II, sent from Duxbury, viz. the mean value of x derived from all the other signals of Class II. An assumed value of (x) was applied to the signals of Class IV., viz. the mean value of x obtained from all the signals of Classes II. and III.

DIFFERENCE OF LONGITUDE BETWEEN BREST AND DUXBURY.

Date.	λ	λ	λ	λ
1870.	CLASS II.	CLASS III.	CLASS IV.	MEANS.
	h m s	h m s	h m s	h m s
Jan. 5		4 24 42.746		4 24 42.746
" 8	4 24 42.893	42.879		42.886
" 9	42.895	42.858		42.876
" 22	42.853	43.016		42.935
" 26	43.101	43.188		43.144
" 28	42.152	43.276		43.214
Feb. 10	42.645		4 24 42.663	42.654
MEAN of Division A				4 24 42.922 ±.051
	h m s	h m s		h m s
Jan. 10	4 24 42.515	4 24 42.498		4 24 42.507
" 14	42.003	42.066		42.034
" 17	42.892	42.977		42.935
" 2	43.038	43.148		42.093
Feb. 9	43.105	43.068		42.086
MEAN of Division B (Rejecting January 14)				4 24 42.905 ±.093

Division A includes all the nights on which the local time was obtained at *both* stations by transits observed on the night of the signals. Division B contains all the nights on which the *Duxbury* time was computed from transits observed on the preceding or following nights. The value of the longitude for January 14 is rejected because on that night the time was computed from observations made on January 11 and 16.

The work of the best seven nights makes $\lambda = 4^h\ 24^m\ 42^s.922$ $\pm.051$

The work of four inferior nights makes $\lambda = 4^h\ 24^m\ 42^s.905$ $\pm.093$

If weights are assigned to these two results inversely proportional to the squares of their probable errors, the final result is $\lambda = 4^h\ 24^m\ 42^s.918$ $\pm.045$

The correction for the difference of the Personal Equation in taking transits between Mr. Dean and Mr. Goodfellow is $-\ 0^s.033\ \pm.012$

The correction for the difference of the Personal Equation in recording cable-signals between these observers is $-\ 0^s.018\ \pm.006$

Applying these corrections, we have $\lambda = 4^h\ 24^m\ 42^s.867\ \pm.047$

This result is computed from 1324 signals of Class II., 2245 signals of Class III., and 52 signals of Class IV., in all 3621 signals; of which about half were negative and half positive, and about half sent from Brest to Duxbury and half from Duxbury to Brest.

MEAN RESULTS OF SIGNALS SENT FROM DUXBURY TO CAMBRIDGE.

Date.	Kind of Signal.	Number.	T_1	$T_1 - T_1''$	$\Delta T_1 - \Delta T_1''$	$\lambda - x$
1869			h m	s	s	s
Dec. 14	Sidereal Clock,	58	1 55	—24.156	134.443	110.287
" 14	Sidereal Clock,	59	2 17	—24.185	134.486	110.301
" 15	Sidereal Clock,	59	3 18	—12.776	123.083	110.307
" 15	Key or Hand,	57	3 27	—12.746	123.108	110.362
" 23	Sidereal Clock,	59	3 21	— 6.290	116.727	110.437
" 23	Key or Hand,	60	3 30	— 6.262	116.737	110.475
" 31	Sidereal Clock,	59	3 55	—12.345	122.741	110.396
" 31	Key or Hand,	84	4 6	—12.323	122.742	110.419
1870						
Jan. 3	Sidereal Clock,	59	3 58	—14.508	124.766	110.258
" 3	Key or Hand,	57	4 6	—14.505	124.774	110.269
" 4	Sidereal Clock,	59	4 31	—15.218	125.330	110.112
" 14	Cable Key, Class III.,	63	3 59	— 9.522	120.695	110.173
" 14	Cable Key, Class II.,	48	4 18	— 9.508	120.689	110.181
" 22	Cable Key, Class III.,	70	4 43	—12.396	122.893	110.497
" 22	Cable Key, Class II.,	74	5 3	—12.373	122.899	110.526
" 24	Cable Key, Class III.,	61	4 54	—13.403	124.027	110.624
" 24	Cable Key, Class II.,	110	5 15	—13.421	124.038	110.617
" 26	Cable Key, Class III.,	68	5 8	—15.382	125.654	110.272
" 26	Cable Key, Class II.,	95	5 28	—15.416	125.691	110.275
" 28	Cable Key, Class III.,	62	5 8	—17.307	127.608	110.301
" 28	Cable Key, Class II.,	112	5 28	—17.297	127.636	110.339
Feb. 9	Cable Key, Class III.,	59	6 14	+ 9.796	100.974	110.770
" 9	Cable Key, Class II.,	61	6 37	+ 9.792	100.961	110.753
" 10	Cable Key, Class IV.,	54	6 32	+10.241	100.244	110.485
" 10	Cable Key, Class II.,	20	6 46	+10.256	100.249	110.505

MEAN RESULTS OF SIGNALS SENT FROM CAMBRIDGE TO DUXBURY.

Date.	Kind of Signal.	Number.	T'	$T' - T''$	$\Delta T' - \Delta T''$	$\lambda + x$
1869			h m	s	s	s
Dec. 14	Sidereal Clock,	59	2 5	—24.081	134.462	110.381
" 15	Sidereal Clock,	59	3 33	—12.734	123.126	110.392
" 15	Key or Hand,	59	3 39	—12.765	123.143	110.378
" 23	Sidereal Clock,	59	3 37	— 6.203	116.744	110.541
" 23	Key or Hand,	57	3 43	— 6.241	116.750	110.509
" 31	Sidereal Clock,	59	4 13	—12.295	122.744	110.449
" 31	Key or Hand,	43	4 18	—12.314	122.744	110.430
1870						
Jan. 3	Sidereal Clock,	59	4 12	—14.478	124.781	110.303
" 3	Key or Hand,	46	4 15	—14.496	124.784	110.288
" 22	M. T. Clock,	50	4 12	—12.355	122.885	110.530
" 24	M. T. Clock,	50	4 26	—13.361	124.014	110.653
" 26	M. T. Clock,	50	4 29	—15.299	125.583	110.284
" 28	M. T. Clock,	50	4 48	—17.240	127.578	110.338
Feb. 9	M. T. Clock,	50	5 33	— 9.790	100.995	110.785
" 10	M. T. Clock,	50	5 25	—10.277	100.225	110.502

DIFFERENCE OF LONGITUDE BETWEEN CAMBRIDGE AND DUXBURY, FROM SIGNALS SENT IN BOTH DIRECTIONS.

Kind of Signal.	Date.	$\lambda - x$	$\lambda + x$	x	λ
Sidereal Clocks.	Dec. 14	s 110.294	s 110.381	s 0.044	s 110.337
	" 15	110.307	110.392	0.042	110.350
	" 23	110.437	110.541	0.052	110.489
	" 31	110.396	110.449	0.027	110.422
	Jan. 3	110.258	110.303	0.022	110.281
	" 4	110.112		0.037	110.149
Key or Hand-signals.	Dec. 15	s 110.362	s 110.378	s 0.008	s 110.370
	" 23	110.475	110.509	0.017	110.492
	" 31	110.419	110.430	0.006	110.425
	Jan. 3	110.269	110.288	0.009	110.278
Cable-Key at Duxbury and M. T. clock at Cambridge.	Jan. 14	s 111.176	s	s 0.009	s 111.185
	" 22	110.512	110.530	0.009	110.521
	" 24	110.619	110.653	0.017	110.636
	" 26	110.274	110.284	0.005	110.279
	" 28	110.325	110.338	0.007	110.331
	Feb. 9	110.761	110.785	0.012	110.773
	" 10	110.490	110.502	0.006	110.496

On January 14, 22, and 24, and on February 9, the time at one or both of the stations was computed from the error and rate of the clocks obtained on other nights, in most cases two or three days intervening. The clock-rates were not constant, and the results, therefore, are not trustworthy. On January 4, the clock error at Cambridge was found from poor observations on only two stars. In the following table the observations on all these nights are rejected.

Date	$\lambda - x$	$\lambda + x$	x	λ
Dec. 14	s 110.294	s. 110.381	s 0.044	s 110.337
" 15	110.334	110.385	0.025	110.360
" 23	110.456	110.525	0.034	110.490
" 31	110.407	110.440	0.016	110.423
Jan. 3	110.263	110.296	0.016	110.280
" 26	110.274	110.284	0.005	110.279
" 28	110.325	110.338	0.007	110.331
" 10	110.490	110.502	0.006	110.496
	Mean of 1672 signals		$\lambda = 110^s.375$ $\pm .021$	

The correction for the difference of the Personal Equation in taking transits between Mr. Goodfellow and Mr. Austin is $-0^s.145$ $\pm .007$

Applying this correction we have $\lambda = 0^h\ 1^m\ 50^s.230$ $\pm .022$

for the difference of Longitude between Duxbury and Cambridge.

Difference of Longitude between Brest and Duxbury $4^h\ 24^m\ 42^s.867$ $\pm .047$

Difference of Longitude between Brest and Cambridge $4^h\ 26^m\ 33^s.097$ $\pm .052$

The Coast Survey Transit-Instrument at the Cambridge Observatory occupies a position which is 28 feet (or .025 of a second) west of the Dome. The Coast Survey Transit-Instrument at Brest stood 413.5 feet (or .41 of a second) west of the Geodetic station on the St. Louis Tower. Reducing the longitudes above given to the Dome of the Observatory and the St. Louis Tower, we have:—

Difference of longitude between Duxbury and Cambridge	$1^m\ 50^s.205$	$\pm .022$
Difference of longitude between Brest and Duxbury	$4^h\ 24^m\ 43^s.277$	$\pm .047$
Difference of longitude between Brest and Cambridge	$4^h\ 26^m\ 33^s.482$	$\pm .052$

In the determination of differences of longitude by telegraphic signals, a correction must be applied on account of the Personal Equation of the observers in noting transits of stars; and when the signals are sent by the cable-lines, another correction is required on account of the Personal Equation in observing and recording signals.

1. What is called the Personal Equation in noting transits is made up of many elements, some of which are personal and others instrumental. When a star has reached a wire of the transit-instrument, the phenomenon does not impress itself upon the eye of the observer instantaneously; then the effect upon the eye must be communicated to the brain; then the will must be aroused so as to send its order to the finger; and this order when sent consumes a measurable fraction of time in travelling through the nerve; there is also another delay in the execution of this order by the muscles and the finger. After the finger touches the key, that must move, contact must be made or broken, the disturbance must go through the local circuit to the electro-magnet which works the pen of the chronograph, and the magnet must act upon its armature. All these various processes require a fraction of time, however small, and their aggregate is the Personal Equation. The Personal Equation, therefore, depends partly on the mental and physical constitution of the observer, and partly upon the instruments through which he operates. If the delay in noting transits was the same at both stations, no error would be introduced into the difference of their local times or into their difference of longitude. Even when the instruments at the stations are strictly the same, the observers are different, and, therefore, the Personal Equations are different, and a correction must be applied on account of the difference of the Personal Equations of the two observers. If it is practicable for the observers to exchange stations with each other for half of the campaign, the difference of their Personal Equations will be eliminated when the separate results of the two parts of the campaign are united into a final mean. Where this cannot be conveniently done, as it obviously cannot be done in operating for transatlantic longitudes, the difference between the Personal Equations of the two observers may be ascertained, at least so far as it is constant, by processes especially devised for the purpose.

The difference of the Personal Equation in time determinations between Mr. Dean and Mr. Goodfellow has been computed from observations made by them at the Cambridge Observatory on three nights, viz. May 13, 17, and 18. On May 13, alternate tallies were taken by them, sometimes one and sometimes the other leading. On May 17 and 18, the first and last tallies were taken by one observer, and the three middle tallies by the other. Twenty-six stars were observed on each of these nights.

The difference of the Personal Equation in time determination between Mr. Goodfellow and Mr. Austin was computed from observations made by the first method on May 15, and by the second method on May 17 and 18.

The following results were obtained:—

DEAN — GOODFELLOW.

1870.	May 13		$-0^s.066 \pm .004$
	" 17		$-0.023 \pm .008$
	" 18		$-0.009 \pm .006$
		Mean	$-0.033 \pm .012$

Mr. Dean records a transit *earlier* than Mr. Goodfellow by .033 of a second.

AUSTIN — GOODFELLOW.

1870.	May 15		$+0^s.129 \pm .011$
	" 17		$+0.163 \pm .015$
	" 18		$+0.144 \pm .008$
		Mean	$+0.145 \pm .007$

Mr. Austin records a transit later than Mr. Goodfellow by .145 of a second.

These results were calculated by the formula

$$(M_1 - M_2) + (F_1 - F_2) \sec \delta = \pm \epsilon$$

in which M_1 represents the mean of the observed threads by the observer who led, and M_2 the mean of the observed threads by the observer who followed; and $F_1 \sec \delta$ and $F_2 \sec \delta$ the corresponding reductions to the mean of all the threads. On May 13 and 15, $F_1 - F_2$ was equal to $-0^s.093$; and on May 17 and 18, $F_1 - F_2$ was equal to $-0^s.060$; Coast Survey Transit No. 5 being used on all the nights.

2. If the electrical current sent through the main line were able to operate a relay-magnet, and through it to control the local circuit and battery, it might record itself upon the chronograph at the place where it is received, as it does at the place from which it is sent. But, in fact, the signals are not thus automatically recorded. The way in which they are recognized and registered is described on page 440. A

fraction of a second intervenes between the real and the recorded arrival of the signal. Various elements enter into this loss of time, partly physiological, and partly instrumental, as in the other case; their aggregate effect is the Personal Equation in noting signals. This is not exactly the same for the two observers at the opposite stations, otherwise it would be eliminated by employing signals sent in both directions alternately. Therefore, it must be determined separately for the two observers, and the proper allowance be made for the difference.

A series of observations was made by Mr. Dean on January 24 and 25, in order to determine his Personal Equation in recording Cable-Signals at Brest. The chronograph was adjusted so that it would make two and one quarter revolutions in a minute of time; one second of time being represented on the chronograph sheet by a distance of three quarters of an inch. A galvanometer, similar to the Thompson galvanometer used in the Cable Office, was adjusted in the *testing-room* of the office. The cable-key remained in its usual place in the *instrument-room*, which is at some distance from the *testing-room*. When this galvanometer and the cable-key were united with a single cell of Minotto's battery, and ten thousand units of resistance coils, in addition to the circuit which passed between the cable office and the Coast Survey Station, and the clock and chronograph, were introduced, the beam of light from the mirror of the galvanometer moved in the same way as when the Second Class of cable-signals came from Duxbury. On January 30, the experiments were modified by placing the cable-key in the *testing-room*, and substituting for the first galvanometer that very one in the instrument-room by which all the longitude signals had been received.

A series of observations was made by Mr. Goodfellow on February 8, 9, and 12, in order to determine his Personal Equation in recording Cable-Signals at Duxbury. The current from a Daniell's battery of three elements was sent through the clock and chronograph of the transit-room of the Coast Survey Observatory, and through the cable-key and the break-circuit keys in the Cable Office. The current from a single cell of Minotto's battery was sent through the same circuit, and the galvanometer used in receiving cable-signals from Brest was shunted so as to make it produce deflections similar to those observed with the cable-signals. The signals proceeded from the battery-room, and the observer who noted and recorded them was placed in another part of the building, out of sight and hearing of anything which occurred in the battery-room. In all these observations made at Brest and Duxbury, to ascertain the retardation in noting cable-signals, these signals were sent through the local circuit which connects the Cable Office with the Coast Survey Station. In order to prevent the interference of the clock-breaks with the passage of these signals, the clock is

switched out of the circuit while the signals are passing, and then introduced for one or two minutes between the different sets of signals, so as to graduate the chronograph sheet into seconds. In this way, the exact time of *sending* and of *noting* each signal would be ascertained. As the transmission time in these experiments is insensible, the difference between the recorded times of sending the signal and of noting it gives the retardation in noting cable-signals.

January 24.			January 25.			January 30.		
Number.	Sums.	Means.	Number.	Sums.	Means.	Number.	Sums.	Means.
10	3.54	s 0.354	10	3.39	s 0.339	10	3.11	s 0.311
10	3.21	0.321	10	3.24	0.324	10	3.14	0.314
9	3.06	0.340	10	3.52	0.352	10	3.38	0.338
8	2.97	0.371	10	3.37	0.337	10	3.16	0.316
10	3.55	0.355	10	3.17	0.317	10	3.29	0.329
7	2.41	0.344	10	3.33	0.333	10	2.95	0.295
6	1.95	0.325	10	3.30	0.330	10	2.75	0.275
6	1.93	0.322	10	3.28	0.328	10	2.99	0.299
9	3.80	0.422	10	3.51	0.351	10	3.19	0.319
10	3.54	0.354	10	3.22	0.322	8	2.33	0.291
10	3.69	0.369	10	3.54	0.354	8	2.25	0.281
10	3.67	0.367	10	3.59	0.359			
			10	3.59	0.359			
			10	3.72	0.372			
			10	3.18	0.318			
			10	3.65	0.365			
			9	2.96	0.329			
105	37.32	s 0.355 ±.005	169	57.56	s 0.341 ±.003	106	32.54	s 0.307 ±.004

February 8.			February 9.			February 12.		
Number.	Sums.	Means.	Number.	Sums.	Means.	Number.	Sums.	Means.
9	2.81	s 0.312	5	1.70	s 0.340	12	3.71	s 0.309
10	3.35	0.335	9	2.62	0.291	12	3.30	0.275
12	3.71	0.309	7	2.19	0.313	12	3.44	0.287
12	3.49	0.291	12	3.58	0.298	12	3.38	0.282
11	3.72	0.338	12	3.69	0.307	12	3.22	0.268
11	3.54	0.322	12	4.11	0.343	12	3.38	0.282
12	3.93	0.327	12	3.68	0.307	12	3.18	0.265
12	3.72	0.310	12	3.51	0.292	12	3.15	0.262
12	3.66	0.305	12	3.66	0.305	12	3.13	0.261
12	3.66	0.305	12	3.74	0.312			
12	3.56	0.297						
12	3.93	0.328						
12	3.47	0.289						
12	3.53	0.294						
12	3.41	0.284						
11	3.17	0.288						
184	56.66	s 0.308 ±.003	105	32.48	s 0.309 ±.004	108	29.89	s 0.277 ±.003

The mean delay of Mr. Dean in recording cable-signals, as computed from three hundred and eighty signals, is $0^s.334 \pm .010$.

The mean delay of Mr. Goodfellow in recording cable-signals, as computed from three hundred and ninety-seven signals, is $0^s.298 \pm .007$.

TRANSMISSION TIME BETWEEN DUXBURY AND BREST.

	January										Feb.	Mean.
	5	8	9	10	14	17	22	24	26	28	9	
Class II.		s 1.174	s 1.100	s 1.129	s 1.085	s 1.063	s 1.154	s 1.116	s 1.090	s 1.081	s	s 1.110 ±.008
Class III.	s 1.111	1.127	1.044	1.056	1.114	1.167	1.164	1.308	1.157	1.174	1.217	1.149 ±.015
Mean.	1.111	1.150	1.072	1.093	1.200	1.115	1.159	1.212	1.123	1.178	1.217	1.132 ±.009

The time lost by Mr. Dean in noting signals at Brest was $0^s.334$, and by Mr. Goodfellow in noting signals at Duxbury was $0^s.298$. If we correct the transmission time for the personal and instrumental delay in recording cable-signals, we have

$$x = 1^s.132 \pm .009$$

$$\text{Correction} \qquad = \tfrac{1}{2}(0.334 + 0.298) = .316 \pm .006$$

$$\text{Corrected value of } x = 0^s.816 \pm .011.$$

The mean transmission time between Duxbury and Cambridge, as obtained from the Arbitrary Hand-signals, is $0^s.010$; from the Cable Key-signals at Duxbury and the Mean Time Clock-signals at Cambridge, is $0^s.009$; and from the Sidereal Clock-signals at both places, is $0^s.037$.

When arbitrary hand-signals were sent from Duxbury to Cambridge, arbitrary hand-signals were returned from Cambridge to Duxbury. When such signals are sent, the key breaks only the main line at both stations, and the current acts through a relay-magnet at both stations. Therefore, by combining the results of these signals when sent in both directions, the transmission time is correctly eliminated. When clock-signals from the Mean Time Clock were sent from Cambridge, it only broke the main line as an arbitrary hand-signal would have broken it, and therefore such signals are properly combined with the signals sent from Duxbury to Cambridge by the cable-key. When sidereal clock-signals are sent in both directions, the results may be compared, so as to eliminate the transmission time and deduce a correct longitude. Inasmuch as, however, in this case, the sidereal clock breaks directly the main line, without

the intervention of a relay-magnet at the sending station, and acts through a relay-magnet at the receiving station, it is evident that the difference of time registered by the two chronograph-sheets will express not only the difference of longitude and the transmission time, but also the uncompensated resistance of the relay-magnet. If the difference of local times which obtains when such signals are sent from Cambridge to Duxbury is subtracted from the difference of local times when such signals are sent from Duxbury to Cambridge, the remainder will represent not merely twice the transmission time, but twice the transmission time increased by twice the resistance of the relay-magnet. This accounts for the large value of the *apparent* transmission time derived from the sidereal clock-signals when compared with the transmission time indicated by hand-signals or by the Mean Time Clock-signals.

The St. Pierre and Brest cables, when combined, make a total length of thirty-three hundred and twenty-nine nautical miles. The total resistance of the cable between Duxbury and St. Pierre amounts to 8980.51 ohms; the total resistance of the cable between St. Pierre and Brest (the greater diameter of the wire overbalancing the increased length) is only 8152.80 ohms. When the resistance of the galvanometer is added to the sum of the above-named resistances, the whole resistance between Duxbury and Brest is found to be 18321.31 ohms. The degree of insulation is expressed by saying that the gutta-percha resistance between Duxbury and St. Pierre is 1,722,700 megohms, and between St. Pierre and Brest 6,204,900 megohms; in all, 7,927,600 megohms. It appears that the transmission time of the electrical signals through this cable is .816 of a second. It would be a hasty conclusion, however, to suppose that the velocity of electricity is 4,080 miles a second, or that it has any velocity in the sense in which we speak of the velocity of a cannon-ball or of the velocities of sound or light. There is an advantage in substituting for the word *velocity* the phrase *transmission-time*, or in defining the velocity of electricity as the distance passed over in *any particular case* divided by the time. The peculiarity of the motion is this: if the distance to be traversed by the electricity had been greater, not only would an additional time have been required for the additional distance, but more time would be required for the first distance. Ohm, in his admirable but too long neglected treatise, published in 1827, under the title *Die galvanische Kette mathematisch bearbeitet*, after treating of the permanent state of the galvanic circuit, takes up the subject of the distribution of tensions in the variable state of the circuit, and arrives at this formula:—

$$u = \frac{a}{2l}\,x + a\,\Sigma\left(\frac{1}{i\pi}\sin\frac{i\pi\,(l+x)}{l}\,.\,e^{\frac{-k'\pi^2 i^2 t}{l^2}}\right).$$

In this equation (u) is the tension at any point, the distance of which from the middle of the circuit is (x); (a) is the tension at the point of excitation; $(2l)$ is the length of the circuit; (k') is the conducting power, divided by a coefficient which expresses the specific electrical capacity of the substance; (t) is the time; (π) is the ratio between the circumference and diameter of a circle; (e) is the Napierian logarithmic base; and (i) is any positive number. M. Gangain, in his commentary upon the treatise of Ohm, which he translated[1] into French, has pointed out the conclusions to be drawn from this formula. He says that it has been established, on the assumption that there was but a single electromotive force brought into play in the circuit, that this force was constant, and that the circuit was homogeneous. In a voltaic pile there are many electromotive forces developed at different points of the circuit; if the resistance of the pile is only a small fraction of the total resistance of the circuit, there will be no sensible error in the supposition that the pile is concentrated in one point, that its resistance is nothing, and that the sum of the electromotive forces is represented by the letter (a) of the formula. When the permanent state of the conductor is established, the value of (u) becomes equal to $\left(\frac{a}{2\,l}\,x\right)$; the term comprehended under the symbol (Σ) (which is obviously smaller as the time (t) increases) having become too insignificant to be regarded. This will happen as soon as $(k'\,\pi^2\,i^2\,t)$ is large, compared with (l^2). Whatever value of (t) may be sufficient for this purpose with any given value of (l), it is evident that, in general, the required value of (t) must be proportional to (l^2). This value of (t) represents the *duration of the propagation* before the permanent state is acquired. It is evident that this value of (t) may be less as the value of (k') is greater. If the velocity of electricity is defined as $\left(\frac{l}{t}\right)$, it will have no determinate value, but may be exceedingly great or exceedingly small according to the distance to be travelled. The passage of electricity is not analogous to the transmission of sound or light, but resembles rather the conduction of heat. This will appear from comparing Ohm's formula with that obtained by Poisson for the conduction of heat along a metallic rod when the two extremities are maintained at a constant temperature.[2]

After Kirchhoff had succeeded in deducing the familiar formulas of Ohm, which express the constant voltaic current, from the principles of statical electricity, he gave his attention to the variable state of the current, and he has obtained expressions[3] for the quantity and intensity of the free electricity at any point of the conductor which

[1] Théorie Mathématique des Courants Electriques, p. 177.

[2] Journal de l'Ecole Polytechnique, XIX. p. 53.

[3] Pogg. Ann. der Physik und Chemie, C. 193 und CII. 529.

admit of two interpretations. One of the factors depends on the length, diameter, and total resistance of the conductor. When this factor is very great, the quantity of electricity tends towards uniformity of distribution as the time progresses, whereas the current approximates towards zero. Moreover, the formulæ, in this case, are analogous to those which represent the propagation of sound in a narrow tube. From this Kirchhoff concludes that two electrical waves are transmitted through the conductor in opposite directions, and with a velocity of 310,765 kilometres (about 193,000 miles) per second. If, on the other hand, the critical factor is very small, a result is reached which shows that the propagation of electricity is analogous to that of heat. In these circumstances there is no more any question about the velocity (in the common sense of the word) of electricity, except that it may be pronounced greater when the conductivity and diameter of the wire are increased.

Almost contemporaneous with these researches of Kirchhoff was an investigation by Sir William Thompson on the "Theory of the Electric Telegraph." He obtained a complex formula for the *potential* at any point of the conducting wire (one end being connected with the battery and the other with the ground), one term of which varies with the time. This term is similar to the variable term in Ohm's formula. Thompson concludes that, though an infinitesimal effect may reach the remote end instantaneously, the time required for the current to reach a *stated fraction of its maximum strength* sufficient to show itself (which may be less as the galvanometer is more delicate) at the remote station will be proportional, in different lines, to the product of the square of the length by the resistance, (electrostatically measured,) and by the electrostatical capacity of the unit of length. Therefore, there is no regular velocity of transmission.

"We may infer that the retardations of signals are proportional to the squares of the distances, and not to the distances simply; and hence different observers, believing they have found a 'velocity of electric propagation' may well have obtained widely discrepant results; and the apparent velocity would, *cæteris paribus*, be the less the greater the length of wire used in the observations."[1] For example, as Professor Stokes has said, if the retardation on the submarine line between Greenwich and Brussels (200 miles in length) is one tenth of a second, the average velocity of the signal is 2,000 miles per second. If a similar cable were extended over a semi-circumference of the earth (about 14,000 miles), the retardation would amount to four hundred and ninety seconds! In this case, the average velocity would be seventy times less, or only $28\frac{1}{2}$ miles per second.

[1] Royal Society Proceedings (London), VII. pp. 382 – 99.

These theoretical considerations will go far towards explaining the apparent contradictions between the results obtained by Wheatstone, Walker, Mitchel, Gould, Fizeau, Faraday, Gaugain, Guillemin, Jenkin, Clark, and many others, who have experimented upon the velocity of electricity, — results which range from 288,000 miles per second to 800 miles per second. No two experiments are properly compared with each other unless a variety of conditions is taken into the account. The enormous velocity obtained by Wheatstone favors the supposition that electricity of high tension (as that which exists in a charged Leyden jar) is endowed with a superior power of transmission. But the experiments of Mr. Latimer Clark[1] (quoted by Faraday) have proved that a voltaic battery might be increased from 31 cells to 500 cells, without sensibly changing the velocity, when the current was sent through 768 miles of copper wire covered with gutta-percha. Moreover, Wheatstone's experiment only proved that the electricity went through less than a mile of wire (in addition to the air spaces where the sparks occurred) *at the rate* of 288,000 miles a second. If many persons have hastily come to the conclusion that electricity would actually move through 288,000 miles of the same kind of wire in a single second, they have made the assumption, which neither theory nor observation warrant, that the velocity is independent of the total resistance of the wire and the length to be traversed. Melloni appears to have adopted this view, as an inference from Clark's experiments, already quoted. He says:[2] "The equal velocity of currents of various tension offers, on the contrary, a fine argument in favor of the opinion of those who suppose the electric current to be analogous to the vibrations of air under the action of sonorous bodies. As sounds, higher or lower in pitch, traverse in air the same space in the same time, whatever be the length or the intensity of the aerial wave formed by the vibration of the sonorous body, so the vibrations, more or less rapid or more or less vigorous, of the electric fluid, excited by the action of batteries of a greater or smaller number of plates, are propagated in conductors with the same velocity." The theoretical examinations which have been made of the problem by Ohm, Gaugain, Kirchhoff, and Thompson all point the other way. They all imply that the whole space travelled by electricity is proportional to the square root of the time. A simple computation conducts Gaugain to the conclusion that Wheatstone's experiment, when interpreted in the light of the best theoretical knowledge, only proved that electricity would take one second to run over 268 miles of similar wire. But Gaugain has evidently gone to

[1] London Philosophical Magazine, 1855.

[2] As quoted by Faraday. Experimental Researches in Electricity, III. 577.

the other extreme, and underrated the velocity. He has reasoned as if, in the experiment, the whole distance traversed was only one quarter of a mile of wire. In fact, it was more than this, and the air-spaces besides. What the equivalent in wire would be for these air-spaces it might not be easy to state with precision. Moreover, the tension of the Leyden jar, as Gaugain is ready to see, was not constant during the discharge, and did not, therefore, conform to the conditions of Ohm's calculations. If, however, the theory must not be *rigidly* applied in opposition to Wheatstone's conclusion, it is sufficiently clear that the enormous discrepancy between the velocity to be deduced from his remarkable experiment and that afterwards revealed by direct observation upon long lines of telegraphic wire no longer exists. Indeed, the corrected velocity from Wheatstone's experiment would be unaccountably *small*, when compared with these later determinations, unless they also received the proper qualification demanded by the same theory.

If an objection is made to the inferences drawn from the mathematical theory of electricity, because this theory is perplexed by difficult questions of analysis, a stronger objection holds against the assumption that the velocity is the same for long as for short distances, as this is sustained neither by theory nor experiment. Undoubtedly, the safest way for determining the velocity of electricity — or, more properly stated, the transmission time — over very long wires is by experiments on these same wires. An experiment upon a very large scale was made by Professor Joseph Winlock, at the Observatory of Harvard College, on the nights of February 28 and March 7, 1869. Signals were sent from the Observatory to San Francisco by one line of wire, and returned to the Observatory by the way of Canada, and the times of leaving and returning to the Observatory were recorded upon the same chronograph-sheet. The time required to pass through this enormous loop of 7,200 miles of No. 9 iron wire, and thirteen telegraph-repeaters, was about two thirds of one second. This result agreed closely with the time obtained from signals sent directly between Cambridge and San Francisco with earth connections, and the transmission time deduced by Mr. Bradford in his computation of the Coast Survey operations for longitude between these two cities. The transmission time in this last case was about two fifths of a second; thus showing that the velocity was nearly the same, whether the electricity discharged into the earth or returned by a loop.

If two wires, without any other difference between them but that of length, are compared, the transmission times are as the squares of the lengths, so that the average velocity is inversely as the length. If the diameters and specific conductivities are unlike, we can substitute equivalent lengths of a standard size and quality, and then

compare these reduced lengths. But other things besides the dimensions and quality of the wire influence the velocity of electricity. Ohm's formula, already discussed, contains a letter which represents the electrostatic capacity of the conductor. But he did not live to see its full significance as revealed in subterranean lines of telegraph. In the very year (1854) when Ohm was seized with a mortal illness, Faraday communicated to the Proceedings of the Royal Institution [1] of Great Britain, some remarkable experiments, made originally by Mr. Latimer Clark, and then repeated, under his inspection, at the Works of the Electric Telegraph Company. One hundred miles of copper wire, covered with gutta-percha, were immersed in water, and when a voltaic current was sent through this circuit, phenomena resulted which were never observed when the wire was surrounded by air. The immersed wire was charged like a Leyden jar, the gutta-percha serving as the dielectric, and the water as the outer coating. In this particular case, the copper wire was one sixteenth of an inch in diameter, and the gutta-percha about one tenth of an inch in thickness. Regarded as a Leyden arrangement, the inner coating, or surface of the copper, contained 8,300 square feet, and the outer coating, or the surface of water in contact with the gutta-percha, contained 33,000 square feet. Experiments were made upon wires covered with gutta-percha, and then enclosed in tubes of lead or iron, or buried in the earth, and with similar results. By connecting the various subterranean wires between London and Manchester, a total length was obtained of 1,500 miles. When galvanometers were introduced at intervals of about 400 miles, they were all under the eye of the same observer in London. If a current was sent into this wire the galvanometers were *successively* affected; the last one only after the long interval of two seconds. Faraday explained this slow transmission through such wires, when compared with the velocity of electricity through air-lines (where no near conductor is present to supply the place of the outer metallic surface in the Leyden arrangement), to the increased electrostatic capacity of the wire under the inductive action, and to the time required for the battery to furnish the additional amount of electricity. Sir William Thompson [2] has proved that the electrostatic capacity of such a wire is equal to $\frac{V k S}{4 \pi r \log \frac{r'}{r}}$, in which ($V$) is the potential, ($k$) the specific inductive capacity of gutta-percha, (S) the surface of the wire, (r) the radius of the copper or the inner surface of the gutta-percha, and (r') the radius of the outside surface of the gutta-percha. Every hundred miles of wire, similar to that used in Faraday's first experiments, had an electrical capacity at least equal to a Leyden battery with 8,300 square feet of coated sur-

[1] Proceedings of the Royal Institution, January 20, 1854.

[2] Papers on Electrostatics and Magnetism, p. 41.

faces separated by a thickness of only $\frac{1}{32}$ of an inch of glass. When a current is fully established and the wire is charged, an immersed cable-line will conduct as well as an air-line. But when the connection is first made with the battery, or first broken, the charge in one case, and the discharge in the other, will travel more slowly in the cable-line than in the air-line; in other words, the time of the variable state of the conductor will be prolonged.[1]

In the cable between St. Pierre and Brest, the inner surface of the gutta-percha (which represents approximately the surface of copper to be charged) amounts to about 700,000 square feet, and the outer surface to about 2,000,000 square feet. In the cable between Duxbury and St. Pierre, the inner surface contains about 100,000 square feet, and the outer surface about 300,000 square feet. Both cables united correspond to a Leyden arrangement, in which one surface has about 800,000 square feet and the other about 2,300,000 square feet. If the dielectric were glass instead of gutta-percha, the equivalent thickness of the glass would be, according to Thompson's formula already given, $\frac{k'}{k}$ multiplied by .043 of an inch for the St. Pierre and Brest cable, and $\frac{k'}{k}$ multiplied by .084 of an inch for the Duxbury and St. Pierre cable; k' and k representing respectively the inductive capacities of glass and gutta-percha. Experiment[2] shows that the ratio $\frac{k'}{k}$ is about $\frac{19}{42}$. The electrostatical capacity of the St. Pierre and Brest cable has been already given as equal to .404 of a microfarad for each nautical mile. Its total capacity is about 1,042 microfarads. The electrostatical capacity of the Duxbury and St. Pierre cable is .358 of a microfarad for each nautical mile, and the total electrostatical capacity is about 268 microfarads. The electrostatical capacity of both cables is about 1310 microfarads.

This total electrostatical capacity of both cables expresses the whole amount of electricity they contain when one end is united to a battery having an electromotive force of one volt, and the other end is put *to air;* that is, is disconnected with the ground. As the forty cells of Minotto's battery used at Duxbury were equivalent to about 33.7 of Daniell's cells, its electromotive force would be equal to about 36 volts, that of a simple Daniell's cell being taken as 1.079 volts. If one end of the united cables is connected to this battery, and the other end is insulated in the air, so that the difference of potential between the inner and outer surfaces of the gutta-percha envelope is equal to 36 volts, the whole amount of electricity required to charge the cable is 47,160 microfarads. The insulation of each knot of cable between Duxbury

[1] See also experiments of W. Siemens, Ann. de Ch. et de Ph., XXIX. 394.

[2] Clark and Sabine's Electrical Tables and Formulæ, p. 67.

and St. Pierre being 2,300 megohms, the total insulation is 3,880,000 ohms. The insulation of each knot of cable between St. Pierre and Brest being 2,405 megohms, the total insulation is 2,000,000 ohms. The total insulation of the gutta-percha is expressed by 1,300,000 ohms; and is, therefore, about seventy-two times greater than that of the conductor. If we suppose Duxbury and Brest to be united by a homogeneous cable throughout the entire length, and suppose also that the remote end of the cable is put to *ground*, if the insulation is perfect, the tension will diminish regularly from its maximum at the battery end down to zero at the ground end. Under these circumstances the charge of electricity it would hold, with the battery of 36 volts, would be one half of what the same cable would contain when it was disconnected from the ground; that is, 23,580 microfarads. In the actual case, the two branches of the cable were very different; moreover, the two cables were not immediately joined, but a Leyden condenser was used at St. Pierre, one surface of which was connected with the branch which extended to Brest, and the other surface with the branch which proceeded to Duxbury.

The current sent into the cable would be, according to Ohm's formula, $\frac{E}{R+r}$. If we substitute for (E) the electromotive force of the battery used, viz. 36 volts, and for (R) the resistance of the battery (which was about 68 ohms), and for (r) the total resistance of the conductor, which was about 8,153 ohms for the St. Pierre and Brest branch, and about 8,980 ohms for the St. Pierre and Duxbury branch, adding also the resistance of the galvanometer, we should have $\frac{36}{9409}$ (or .003826 of one veber) for the strength of current between Brest and St. Pierre, and $\frac{36}{10236}$ (or .003517 of one veber) for the strength of current between St. Pierre and Duxbury. When signals were sent from Duxbury to St. Pierre, and then repeated by the condenser between St. Pierre and Brest, 3,517 microvebers flowed into the circuit every second between Duxbury and St. Pierre, and 3,826 microvebers between St. Pierre and Brest. When signals were sent from Brest to St. Pierre and then forwarded by the condenser to Duxbury, the number of microvebers circulating every second in the two branches was less than the numbers just mentioned in the ratio of about 253 to 337, since the battery used at Brest was smaller than that which operated the Duxbury end in this ratio. This maximum strength of the current would not be attained, throughout the circuit, until after connection with the battery had been maintained for an indefinitely long period of time. But ninety per cent of the maximum would be reached in the time allotted to signals of Class IV., and a large fraction of the maximum even in the times assigned to Classes II. and III. For the battery would supply the 18,756 microfarads required to charge fully the longer branch of the cable in five or six seconds, and the 4,824

microfarads for the shorter branch in less than two seconds. As the whole transmission time over both cables amounted to less than one second, it is evident that the end remotest from the battery must have begun to discharge when the cable was imperfectly charged. This might be expected when it is considered that a part of the electricity which constitutes the charge is free to move. As the transmission times for different cables are proportional to the products of the total electrostatical capacities multiplied by the total resistance, the transmission time between Duxbury and St. Pierre would be about one fourth (more exactly $\frac{10}{36}$) of the transmission time between St. Pierre and Brest. Of the whole transmission time between Duxbury and Brest, four fifths (or .639 of a second) would belong to the larger branch, and one fifth (or .177 of a second) to the shorter branch. The former was traversed at an average rate of about 4,000 nautical miles per second, the latter at an average rate of 4,230 nautical miles per second, the whole distance having been passed over at the average rate of 4,080 nautical miles per second.

The transmission time obtained by Dr. B. A. Gould in 1866, for the passage of signals between Valencia and Newfoundland, was about three tenths of one second. If we apply Thompson's formula, viz. that the time which elapses before the current reaches a stated fraction of its maximum strength is proportional to kcl^2 (or the total electrostatical capacity (cl), multiplied by the total resistance (kl),), we have the ratio of the times for signals to pass between Valencia and Newfoundland, and between Brest and St. Pierre, expressed by the fraction $\frac{(1852)^2 \times 3.89 \times .353}{(2580)^2 \times 2.93 \times .429} = \frac{47}{83}$ nearly. When the transmission time observed between Brest and St. Pierre (viz. .639 of one second) is multiplied by this ratio ($\frac{47}{83}$), the product is .36 of a second. Hence there appears to be a satisfactory agreement between the velocities of transmission, as deduced from the longitude campaign of 1866 upon the Anglo-American cable, and the longitude campaign of 1869 – 70 upon the French cable, when the two are reduced to the same standard of length, conductivity, and electrostatical capacity.

Mr. Varley[1] made experiments with a battery, varying from twelve to thirty-six of Daniell's cells, upon a cable coiled in a mass, and upon lengths of 150, 300, and 450 miles, and found the transmission times independent of the force of the battery, but proportional to the squares of the lengths of cable introduced into the circuit. He also experimented upon 270 and 540 miles of submerged cable between Dunwich and Zandvoort, and found the time on the shorter length to be only one quarter of what it was with the double length. Jenkins[2] has described some observations which he

[1] Proceedings of the Royal Society, London, XII. 211. [2] Proceedings of the Royal Society, London, XII. 198.

made upon the Red Sea cable while it was coiled in iron tanks, and he found that, although the electromotive force had no effect upon the velocity, the rate of transmission was inversely as the square of the length. When Quetelet and Airy despatched signals between Greenwich and Brussels, in order to ascertain the difference of longitude, the observers exchanged stations with each other, during a part of the operation, for the purpose of eliminating the Personal Equation. The length of the line was 270 miles, of which about 180 miles consisted of a subterranean coated wire. The transmission time (one tenth of a second [1]) was comparatively large for so short a line. This was due, doubtless, to the great resistance and electrostatical capacity of the core, but I do not possess sufficiently accurate knowledge of its character to calculate their precise values. The same difficulty applies to Faraday's experiment [2] in which he obtained two seconds as the transmission time of electricity in passing over 1,500 miles of coated wire. Moreover, the galvanometer which he used was not the same as those employed on the transatlantic cables. Varley experimented upon 1,600 miles of the same wire, in 1854, and obtained a transmission time of three seconds, the arrival of the electrical current being recognized by its *chemical* effect.[3] Mr. Latimer Clark found the transmission time over 768 miles of coated wire to be two thirds of one second. In this last case the dimensions of the copper wire and the core are given. The formula, by which the times are compared, when the lengths, diameters, and electrostatical capacities of two lines are known, would lead us to expect in Mr. Clark's experiment a time only one half as great as the transmission time between Brest and St. Pierre. In reality it was quite as large; but no allowance could be made for the difference of galvanometers, or the specific conductivities of the wire. It should also be kept in mind, that in some of the various experiments made upon the transmission time of the current, the conducting wire was simply covered with gutta-percha, whereas, in other cases, it had, outside of the gutta-percha, a protecting armor of iron. Now it is easily conceivable, as was suggested by Mr. Varley, that magnetic induction in the iron would not be without its influence on the rate of transmission. Moreover, the theory of Ohm was framed before the discovery of the *extra* current induced in the conductor, and could not, therefore, have taken into its account any effect which that may exert upon the propagation of the primitive current from the battery. Furthermore, it should be remembered that, thus far, I have supposed that the wire, if a naked one, was uninfluenced by the surrounding

[1] London Athenæum, 1854, p. 54.

[2] Experimental Researches in Electricity, III. 512.

[3] Proceedings of the Royal Society, London, XII. 211.

air; or, if it was a cable wire, coated with gutta-percha, that it suffered no loss of electricity from the surrounding water; in other words, that, in both cases, the insulation was perfect, so that there was no leakage to the electrical current. The influence of this leakage, at least so far as the naked wires are concerned, has been carefully studied by Gaugain,[1] and the deductions from Ohm's theory have been tested by nice experiments, and with one curious result. Gaugain designates the time required by the current to reach its *highest limit* of tension at any point as the *absolute duration of the propagation*, and he calls the time which elapses before the current attains a definite percentage of this maximum the *relative duration of the propagation.* Now, when the disturbing effect of the air is taken into account, in consequence of its imperfect insulation, Ohm's theory indicates that the absolute duration of the propagation would increase *faster* than in proportion to the square of the length of the conducting wire, and that the relative velocity of propagation would increase more *slowly* than the square of the length. The absolute duration of the propagation is of no practical importance as it is indefinitely long in any case; and there can be no question as to its comparative values. But the law in regard to the relative duration of propagation was tested experimentally as follows. Two threads of cotton were taken, each $1^{m}.65$ in length. When tried separately, the transmission time was on the average eleven seconds. If they were placed end to end, so as to double the length, the transmission time was four times larger, or forty-four seconds. In these experiments the loss by the air was comparatively insignificant. Gaugain tried, next, two threads of silk, each four metres in length, so that the loss by the air might be sensible, compared with their own imperfect conducting power. The transmission time with the united lengths (eight metres) was only three times as great as when either was tried alone. He obtained the same result with two threads of cotton, each only one metre in length, when the current was diverted laterally by means of three pieces of silk edging symmetrically placed. The battery used in all the experiments consisted of 630 elements, of which 140 were those known as the *couronne de tasses*, and 490 were those of Pulvermacher.[2] These results accord with Ohm's more general expression for the tension, in which the influence of the air is not neglected, viz.: —

$$u = \frac{a}{2}\left(\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta l} - e^{-\beta l}}\right) + a \,.\, e^{-k'\beta^2 t} \sum \frac{i\pi \sin \frac{i\pi(l+x)}{l}}{i^2\pi^2 + \beta^2 l^2} \,.\, e^{\frac{-k'\pi^2 i^2 t}{l^2}} .$$

The formula, complicated as it is, holds good only on the presumption that the disturbing influence of the air is uniform throughout the whole extent of the circuit.

[1] Annales de Chimie et de Physique, 3 S., LX. 326.
[2] Annales de Chimie et de Physique, 3 S., LXIII. 201.

Gaugain despairs of realizing this essential condition upon any long telegraphic line, and he therefore concludes that it is difficult, if not impossible, to verify these laws on such a scale of magnitude. Guillemin has made many experiments upon the telegraphic lines in France. In Gaugain's experiments upon short, imperfect conductors, it appeared that the rate of propagation was no greater with strong batteries than with weak ones; and this has been the general verdict in experiments upon air and cable lines of telegraph. But Guillemin found that when the number of elements in his battery was doubled, the transmission time was diminished about ten per cent. To ascertain the rate of transmission, he employed lines varying in length from 280 kilometres (174 miles) to 1,004 kilometres (624 miles). The rate of transmission was much greater than in the simple proportion of the length, but decidedly less than in proportion to the square of the length. With 60 elements of Bunsen the transmission time over 354 miles of iron wire 4$^{mm.}$ (or .157 of an inch) in diameter was only .020 of one second. Therefore the electrical disturbance travelled over this particular distance at the rate of 17,710 miles in one second.[1] It should be understood that generally the transmission time has been ascertained by the arrival of the current in sufficient force to affect a galvanometer; but in Gaugain's experiments the time required for the conductor to acquire a stated tension was determined by means of a delicate gold-leaf electroscope. The resistance *per knot* of the Anglo-Atlantic cable is less than that of the cable between Duxbury and St. Pierre, and very much less than that of the cable between St. Pierre and Brest, partly on account of difference of dimensions and partly because of differences of pressure and temperature where they are laid. Nevertheless, the total insulation of the Anglo-Atlantic cable is about 1,316,000 ohms, and that of the whole length between Brest and Duxbury 1,300,000. I infer, therefore, that little or no allowance need be made, in consequence of any large difference of leakage, in the comparison which has already been presented of the transmission times over these two long cable-lines.

We may hope that the longitude campaigns of the United States Coast Survey may have been useful, not only in determining differences of longitude, but also in throwing light on the delicate problem of the transmission of electrical disturbances. As we have pursued this discussion we have felt our obligation to Professor Winlock of the Harvard College Observatory, and to Mr. Dean and Mr. Goodfellow, and their assistants in the United States Coast Survey service, for the intelligence and energy with which the observations were planned and executed. But the best observations, however numerous, will not bring out a satisfactory result, unless they are skilfully

[1] Annales de Chimie et de Physique, 3 S., LX. 385.

handled by the computer. That this part of the work has been done laboriously and wisely by Mr. Lucius Brown, the result itself is sufficient to demonstrate.[1]

Electrical Tests of Atlantic Cables as given by Clark and Sabine.

Cable	Anglo-Atlantic.	Anglo-Atlantic.	French-Atlantic.	St. Pierre-Duxbury.
Date	1865	1866	1869	1869
Length (in knots)	1896	1852	2584	749
Diameter of Copper	.147	.147	.168	.087
Diameter of Core	.467	.467	.470	.282
Resistance of Conductor, in Ohms, per knot	4.27	4.20	3.16	12.03
Its specific Conductivity [2] (pure copper being 100)	93.09	94.63	94.33	92.63
Resistance of Dielectric in Megohms per knot [2]	349	342	235	266
Electrostatic Capacity per knot in Microfarads	.3535	.3535	.4295	.3740
Resistance of Conductor per knot (when laid) in Ohms[3]	4.01	3.89	2.93	11.12
Resistance of Dielectric per knot (when laid) in Megohms [3]	2945	2437	5200	2910

[1] In the Preface to the *Electrical Tables and Formulæ*, compiled by Latimer Clark and Robert Sabine, it is said: "It may be mentioned that, by common consent, the value at first assigned to the *farad*, as expressing the unit of capacity, has now been assigned to the *microfarad*; this was done to preserve the unity and simplicity of the system." What is called a *farad*, on page 442 of this memoir, is of the same value as the *microfarad*, introduced on page 471. It will be observed that many of the numbers given on page 442 differ from those attached to the French cable on pages 252 – 3 of the work just quoted, and generally for obvious reasons. In the computations I have adopted the more appropriate numbers which belong to the cable, *after it was laid.*

[2] At 24° Centigrade.

[3] Irrespective of Temperature and Pressure.

ERRATA.

Page 439, line 8 from the bottom, third word in the line; for *arc* read *time.*

Page 445, in last formula; insert $+$ between ω_0 and the fraction.

Page 455, in values of λ, Class II., Jan. 14: in set 4, *for* 42.991 *read* 41.991; in set 6, *for* 42.979 *read* 41.979.

Page 457, in value of λ, Class II., Jan. 28; *for* 42.152 *read* 43.152.

In the same table, on the right hand column, Division B, opposite Jan. 24, *for* 42.093 *read* 43.093; opposite Feb. 9, *for* 42.086 *read* 43.086.

Page 458, in values of $(\lambda - x)$, opposite Jan. 14; *for* 110.173 *read* 111.173, and *for* 110.181 *read* 111.181.

At the bottom of same page, in the column $(T' - T'')$, opposite Feb. 9, *for* -9.790 *read* $+9.790$; opposite Feb. 10, *for* -10.277 *read* $+10.277$.

Page 464, in line of means in the table: under Jan. 14, *for* 1.200 *read* 1.100; under Jan. 28, *for* 1.178 *read* 1.128.

The errors above named are simply misprints, which are made obvious by the context, and in no way affect the results.

On page 452, the first normal equation for obtaining the weight of $\Delta\theta$ should be $\Delta\theta + .616a - \frac{1}{24}$ (or .042) $= 0$. Hence the value of p (or the weight of $\Delta\theta$) is 19.86. Therefore, $r_0 = \frac{r}{\sqrt{p}} = 0^s.027$.

On page 453, according to the notation elsewhere used in the memoir, all the values of ΔT_0 for Cambridge and the *Hourly Rate* should have the sign *minus* prefixed to them. It should be understood that the *Hourly Rates* on pages 453 and 454 are the approximate values used in computing (ω_0), and not the corrected rates used in calculating the longitude.

The values found for the differences of longitude are not affected in any way by these corrections.